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ROYAL METEOROLOGICAL SOCIETY

Editor: J. S. SAWYER

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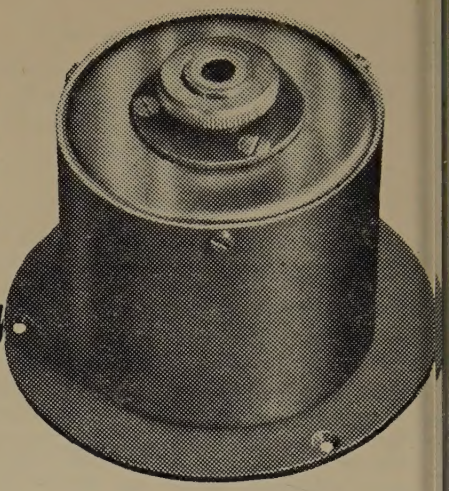

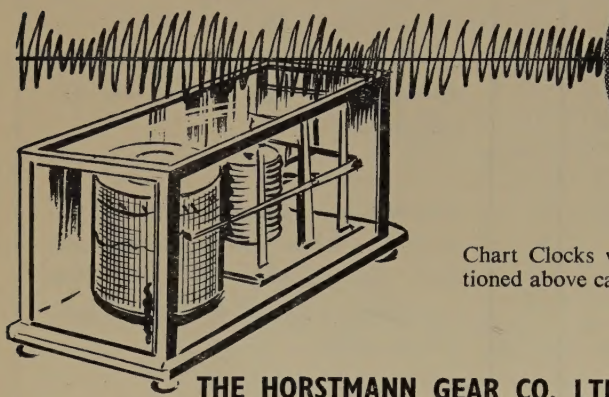

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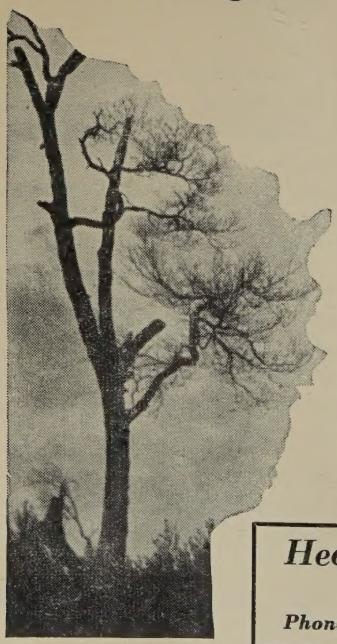

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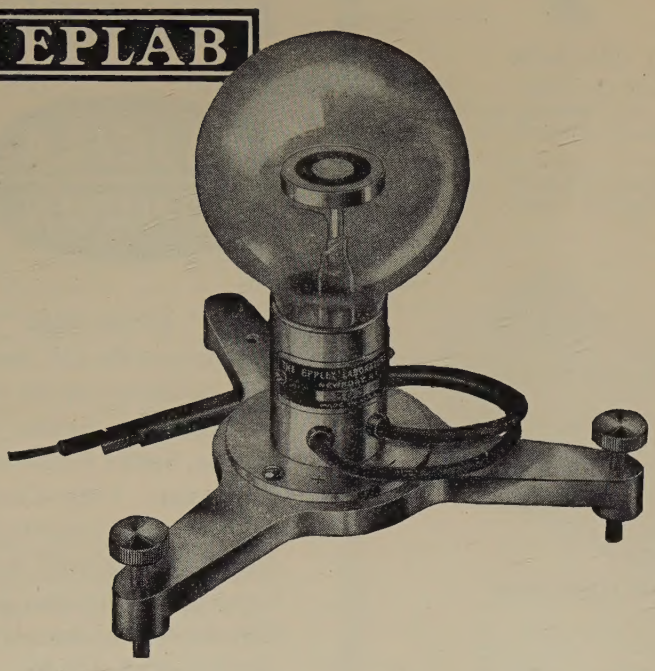

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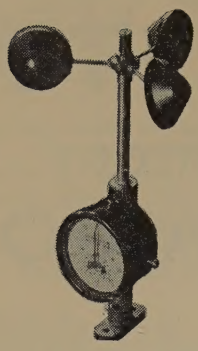

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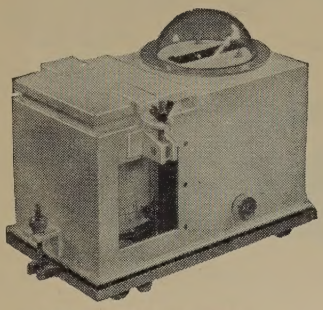

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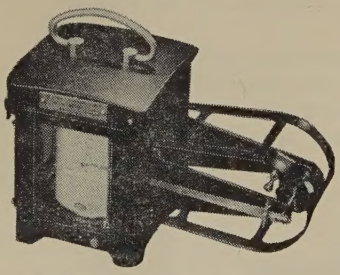

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Temperature fluctuations and trends over the earth

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A.R.D.E., Ministry of Supply, London

(Manuscript received 13 June 1960)

SUMMARY

The annual temperature deviations at over 400 meteorological stations are combined on a regional basis to give the integrated fluctuations over large areas and zones. These are shown in graphical form, and it is concluded that a solar or atmospheric dust hypothesis is necessary to explain the world-wide fluctuations of a few years duration. An important change in the relationships of the zonal fluctuations has occurred since 1920. The overall temperature trends found from the data are considered in relation to the homogeneity of recording, and also to the evidence of glacial recession in different zones. It is concluded that the rising trend, shown by the instruments during recent decades, is significant from the Arctic to about 45°S lat., but quite small in most regions below 35°N. and not yet apparent in some. It is thought that the regional and zonal distribution of recent climatic trends is incompatible with the hypothesis of increased solar heating as the cause. On the other hand, the major features of this distribution are not incompatible with the hypothesis of increased carbon dioxide radiation, if the rate of atmospheric mixing between the hemispheres is a matter of decades rather than years.

1. INTRODUCTION

The many books and articles published during the past century or more on the subject of climatic change testify to its unflagging interest, and, no doubt, also to the number and variety of the theories which have been advanced to account for such changes. Some of these theories, and much evidence of change, are discussed in *Climatic change* (Shapley 1953), and also by Lysgaard (1949) who has assembled a bibliography of no less than 500 items on this subject. Another source of much useful data and discussion of recent events in this field will be found in Ahlmann (1953). Recently Kraus (1960) has suggested that both ancient and modern climatic fluctuations are most readily explained on the basis of changes in the atmospheric cooling rate by long-wave radiation.

The present paper is intended to illustrate the fluctuations and trends of temperature which have occurred in various parts of the world during recent decades, and to suggest possible conclusions which may be drawn from them. This subject, however, is a vast one, and only too easily submerged in an ocean of repelling statistics, unless firm measures are taken to reduce the mass of data into a form which eliminates distracting or irrelevant detail, whilst emphasizing the large-scale features. With this object in view an attempt is made to illustrate the data in a manner which is not too uninviting for the ordinary reader's attention.

As regards the longer-period fluctuations, or trends of climate, which are, no doubt, of the most general interest outside the specialist field, it may be said that the extension of the data for another 10 to 20 years lends support to the conclusions reached in earlier studies of this nature by Lysgaard (1949) and Willett (1950); namely, that a gradual rise in the temperature climate has been in progress for several decades, with a maximum rate in higher latitudes of the Northern Hemisphere, and relatively small rates in most other regions. Temperatures, however, still appear to be quite stable in a few limited areas, amongst which the Black sea – Caspian region, most of Australia, and the far south Atlantic Ocean, are the most notable.

Some suggestions concerning the origin of these climatic trends will be found in the final section of this paper, but this is a difficult subject: by long tradition the happy hunting ground for robust speculation, it suffers much because so few can separate fact from fancy.

2. The data

Continuous temperature records of long duration are available nowadays for most parts of the world, although wide gaps still exist in some regions – notably the southern oceans and the Antarctic. Much experience is necessary, however, in handling these data because of the extreme variability in accuracy of the recordings over long periods, and it is always advisable to take into account any historical information concerning them before accepting the published figures. Some general remarks on the methods used for assembling the data presented here will be found in the following sub-sections.

(*a*) *Assembly of data*

In order to limit the size of this investigation to reasonable proportions, and to obtain results of the most general significance, only mean annual temperatures are used. About 600 of these records have been examined, and the combined differences from average used to compute the fluctuations in all countries or regions where there are several series of long duration. When weighted according to area, the averages in these regions give the zonal or world-wide fluctuations, apart from the blank areas in the south previously mentioned. Because of the amount of computation involved by this procedure, and in view of the possible use to other investigators, it has been thought worth while to include the annual deviations for the zones as appendix tables to this paper. The majority of these annual temperatures have been obtained from *World Weather Records* (1927, etc.), and the historical notes included therein have proved most valuable in assessing the homogeneity of many series. Some few score others have come from a variety of sources, too numerous for mention here.

The method recommended by Willett (1950) of taking back successive differences from a late period, has been used to bring in stations which do not cover the whole of the standard average period required to place regional fluctuations on a uniform basis. Any period of 30 or more years is suitable for this purpose, but it is preferable to use a middle period, such as 1901-1930, which is covered by a very large number of stations. The difference from, say, 1921-1950, in particular districts or regions, may then be used to bring in shorter series, after checking back their decadal fluctuations with neighbouring stations.

(*b*) *Assessment of accuracy*

As might be expected from the number of series used, and the random nature of most of the errors, the latter will hardly affect deviations for large areas or zones which are only taken to the nearest one-hundredth of a degree, (i.e., one or two figures in almost all cases). There are, however, certain well-known sources of error which cannot be classed as random because they may similarly affect a number of stations, and thus seriously influence the temperature trend recorded for wide areas. Of these the most common is known as ' Urban effect ' – the rising trend usually found when an observing site becomes sheltered by new buildings, or a large increase of population in the near neighbourhood. In certain countries where large numbers of temperature series have been published in compact form (e.g., ' Climate records of Japan and the Far East area,' 1954) it has been possible to compare the trend shown at both city and rural stations. Thus it was found that the 30-yr change, 1921-1950 minus 1891-1920, averaged for the nine largest cities in Western Europe, was 0·42°C, whereas the average of this change at 14 rural-type stations, covering approximately the same area, was 0·36°. Similar comparisons in other regions, over a variety of periods, usually showed a small excess of rising trend at the urban stations,

mounting to from 0·03 to 0·1 on a 30-yr change. Thus when using a mixture of urban nd rural sites, as is typical of those given in *World Weather Records*, one must bear in mind that average temperature trends may be too large by a factor of around $U/400\,n$ °C/yr, where U/n is the proportion of urban sites used for the average. This correction, however, has not been used here owing to compensating factors in many regions.

Almost as common as urban increase is the opposite effect on temperatures caused by 'improved exposure' of the instruments. This is especially noticeable in some of the very old series, and also in some nineteenth-century readings from tropical climates, when the importance of radiation effects was becoming recognized and modern screens were substituted for inefficient, or badly placed old types. Also under the heading of improved exposure comes the removal from city to airport sites, so frequent of late decades, which may give a sharp drop in recorded temperature. Corrections for these exposure changes are, however, often given, or used, in the published data, and, in any case, are less difficult to detect than the urban effects. Other sources of error have been fully documented by Mitchell (1953). Most are small and of a random nature, provided reasonable inspection of the stations is maintained.

Apart from rejecting some few dozen obviously erratic or over-urbanized series, the policy here has been to avoid introducing any subjective factor into the data, and to rely on numbers to minimize the minor exposure changes. The results of various tests for the homogeneity of all the stations used are as follows :

(1) Known from history, site, and intercomparison to be very reliable – 16 per cent.

(2) Appear from history and cross-checking to be quite reliable – 58 per cent.

(3) Towns stations showing marked urban increase – 8 per cent.

(4) Doubtful, and probably unreliable records – 18 per cent.

From this analysis it appeared that about three-quarters of the temperature records as published are reasonably homogeneous, in the twentieth century at least, and a few of those in Europe for 100 years, possibly more. In addition many of the doubtful series are reliable enough after a break in an early period.

(*c*) *Presentation of data*

In order to present this mass of data in an acceptable form some smoothing is necessary, because annual deviations are unattractive for illustrating a long series : the significant oscillations are lost to the eye in a jumble of confusing detail. Here a 5-yr smoothing* has been used to date the short-period fluctuations – a procedure which gives graphs similar to the 5-yr moving average, but with better appearance and dating of the short fluctuation. For illustrating longer variations, or temperature trends in the zonal averages, a 20-yr moving average seems to be quite acceptable, although this average usually gives an ugly and distracting graph for a single station in higher latitudes, owing to the erratic incidence of cold or warm years.

3. Discussion of figures

For brevity, the sections are sub-divided, but only a few of the more outstanding points can be discussed here. For the rest, the Figures must speak for themselves.

(*a*) *The reality of temperature fluctuations integrated over large areas*

To begin we may consider the fluctuations in various tropical regions, as shown in Fig. 1, because doing so will bring out a point of fundamental importance in an investigation of this kind; namely, that the averages of temperature from a few well-distributed

* $Ts = \frac{1}{9}(T(n-2) + 2T(n-1) + 3Tn + 2T(n+1) + T(n+2))$ where n is the date-year on the graph.

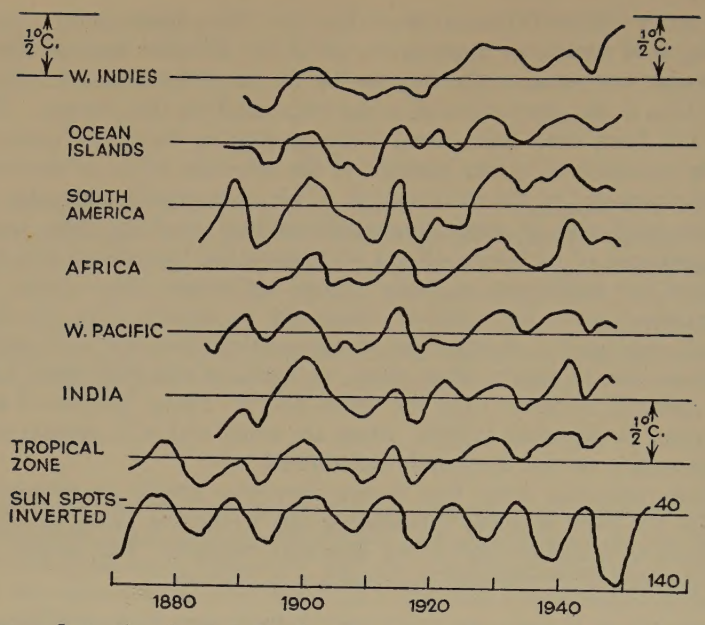

Figure 1. Temperature fluctuations in tropical regions, 5-year annual mean departures from mean 1901-1930

stations, (in this case 10 to 20 stations for areas of 4 to $7{\cdot}10^6$ km^2), do in fact record rea fluctuations over great regions : otherwise it would not be possible to account for the remarkable concurrence in time of those fluctuations shown by the graphs in Fig. 1 The latter supply definite observational evidence on a point which is by no means above controversy amongst those who are unfamiliar with this type of data.

(*b*) *Regional fluctuations in the tropical zone*

Regarding the fluctuations themselves, the impression is certainly given that they are caused by some overriding factor such as solar variation, or atmospheric transparency. Undoubtedly there is a striking correspondence between them and the inverted sun-spot curve, (bottom graph in Fig. 1), from 1875 to 1920. Since then, however, the phase has apparently reversed. On the other hand, if the fluctuations are caused by pulses of the trade winds or general circulation – in the sense that a warm period represents a tendency to stagnation, soon corrected by the induced pulse of advection from colder latitudes – it seems remarkable that they should occur at approximately the same time all round the tropical zone. Solar fluctuations would seem necessary to set the time scale.

In the case of the largest tropical fluctuations, about 1900, 1915 and 1940, annual temperatures rose and fell almost simultaneously at the great majority of the stations. Even with a greatly increased number of stations in the last decade, almost all showed a significant fall from 1941 to 1943. Another point of interest, and possible significance to a solar origin for these fluctuations, is that the high-level station of Quito, on the Equator, regularly records all the warm and cool years given by the zonal average for the Tropics. Thus the latter gives 1893 and 1917 as coolest years, and 1941 as warmest in the 70 years 1881 to 1950. These were also the ' record ' cold and warm years at Quito.

(*c*) *Regional fluctuations in the temperate zones* (*Fig. 2*)

Only a few widely separated temperate regions have been shown in Fig. 2, because it was found that if many were plotted on one small-scale figure the result was a confusing jumble of lines, too unattractive for anything but a hasty glance. Even so the contrast

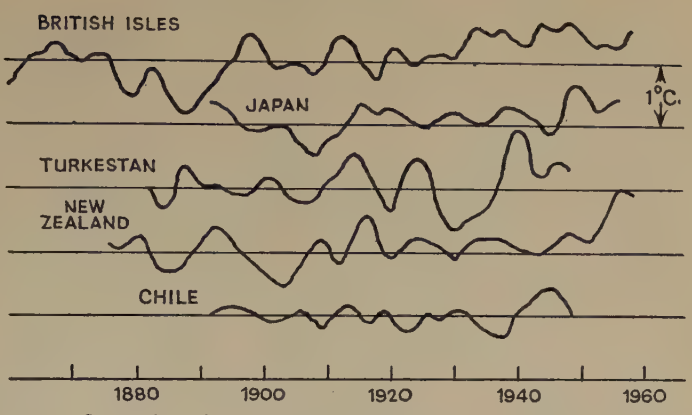

Figure 2. Temperature fluctuations in parts of temperate zones; 5-year annual mean departures from mean 1901-1930.

Between Fig. 1 and 2 is very striking. In place of the orderly progression of oscillations in different parts of the tropics, we find a chaos of vigorous fluctuations in the temperate regions, sometimes directly opposed for long periods, at others running parallel over many years. These variable fluctuations in the temperate zone certainly do not lend themselves to a solar explanation. As a rule they can be related to semi-permanent displacements, in the average ridge and trough structure of the zonal westerlies, although there are difficulties where the longer-lasting fluctuations are concerned. (For example, Petterssen (1949) finds that the higher average temperature in the West Greenland region during the 1930's cannot be explained by increased advection from the south).

(*d*) *Temperature fluctuations integrated for the zones* (*Fig. 3*)

The annual deviation for the zones (appendix tables), has been obtained by combining each region, after weighting them according to the area represented. In the case of isolated stations such as oceanic islands, an equivalent area of 100-degree squares has been allotted to bring them into the zonal average. These deviations for the whole zone, however, are only computed to the nearest 0·01°, or two figures at most, and experience shows that it makes hardly any difference whether the stations are grouped into large regions of roughly equal area, or into many small districts of about the same size.

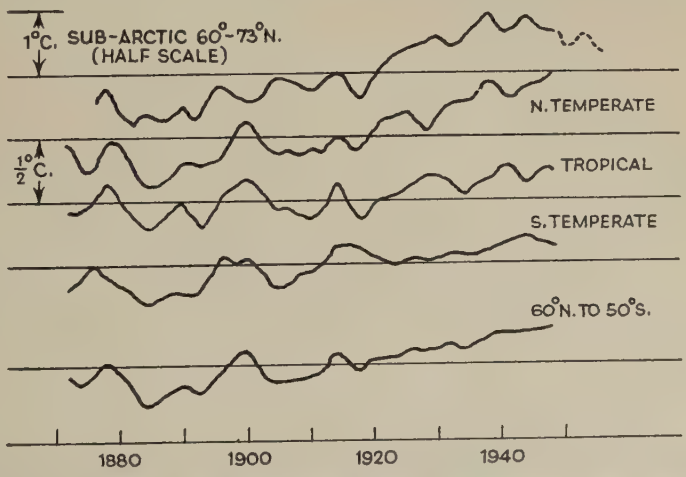

Figure 3. Temperature fluctuation integrated for the zones; 5-year annual mean departures from mean 1901-1930.

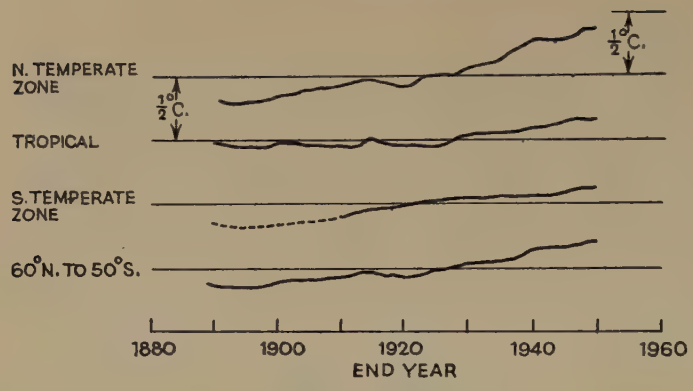

Figure 4. Temperature trends integrated for the zones; 20-year moving average departure from mean 1901-1930.

The zonal graphs of Fig. 3 demonstrate that all the chief fluctuations from 1875 to 1915 occurred simultaneously in both temperate and tropical regions. Since then the picture is more confused, with the minor fluctuations taking a different course in each zone; and, from about the same time, a marked rising tendency has occurred in the northern zones, with a small but significant rise in the Tropics. From about 1900, if not before, these fluctuations are undoubtedly a fair approximation to the actual variations of temperature over a major part of the earth's surface, and, as such, they must be taken into account when considering how recent changes in northern latitudes came about. In doing so it is probably necessary to assume quite separate causes for the short fluctuations than for the overall rising tendency during recent decades.

The fluctuations in the sub-Arctic zone, from approximately 60° to 73° latitude, have not been included in the continued zonal average (bottom graph, Fig. 3), because they are very large, and there are no equivalent areas represented in the Antarctic to counterbalance them. In any case they cover such a small area, compared with the other zones, that their inclusion would make little difference to the overall zonal average. Scattered data for the 1950's show that the big rise in this zone during the 1930's and 1940's has receded, for the time being at least, with averages in Alaska and north Scandinavia nearly back to those for 1901-1930. There is, however, little indication of a similar recession in temperate latitudes, for the 1950's have given higher averages than previous decades at several places, of which the most notable are eastern North America, Japan, Egypt and New Zealand.

4. Trends in zonal temperatures (Fig. 4)

Before considering these zonal trends a reminder is necessary that the nineteenth-century averages are less reliable than those for later decades. This is especially so for the Southern Hemisphere owing to the small number of stations available at that time, and it also applies to the early tropical data, although there are many more stations, and a smaller proportion of urban sites in the latter. Some general remarks on accuracy, with special reference to 'urban effect,' have already been made in sections 2 and 3, and in the following sub-section an indication is given of where the zonal averages are likely to be most reliable.

(a) *Reliability of the zonal temperature trends*

In all the land areas of the north temperate zone, with the exception of east Central Asia, there are reasonable numbers of published temperature series for cross-checking back 60 to 70 years. In Western Europe to about 20°E. long., U.S.A. and Japan, ample

cords have been available for very close checking, using wherever possible, the ' rural be ' stations. For U.S.S.R., it is reported that thermometer screens are established n an open place outside the town,' (*Notes, W.W.R.*, 1921-1930), and examination of the -yr deviations at 50 stations in that large country certainly suggests that very few of em have suffered from urban effects, or sudden exposure changes. Quite a large propor- n of the many coastal and island stations used to represent the northern oceans have exceptionable sites, but the central North Pacific for nearly 100° longitude is a blank ea, which might be expected to have a somewhat smaller trend than the continents. t the Azores the trend for 1891-1957 appears to be only about half the zonal average, t further north it is considerably greater). It may be noted that the 20-yr deviations r the north temperate zone 1900 to 1940, as given in Fig. 4, are closely similar to those mputed from Willett's (1950) curves for the zone between 30° and 60°N. lat., although out 4 times as many stations have been used here.

In the tropical regions a marked feature is the smallness of the 10 or 20-yr deviations compared with the other zones. There is a close cover of stations available since 1881 r India (Pramanik and Jagannathan 1954), but many of the individual series are erratic r northern standards. The vast tropical oceans are only represented by a dozen or so ng series, which, however, include a few of the highest class, such as the Royal Alfred bservatory at Mauritius and, most probably, Apia, should be included with Honolulu this class. These show the same tendency as the zonal average for the temperature to eep up a few tenths of a degree in the later decades, but on the Mexican plateau, and at e high Andes stations of Quito and La Quiaca this tendency is much more marked. could, no doubt, be related to recent recession of glaciers on the high tropical mountains amirez 1958, Spink 1949, Charnley 1959, etc.).

Little can be said about the south temperate zone. The uncertainty of the earlier ta has already been mentioned, and later it can only give the trend for the very limited nd areas. No doubt very stable temperatures prevail over the great southern oceans om one decade to another, and it would be quite legitimate to doubt the small upward end for this zone shown by the average in Fig. 4, were it not for the evidence of con- derable glacial recession in the Southern Alps and Andes (Harrington 1952, Broggi 945, and several others). This latter suggests that the trend found for the land areas this zone is reflected in a similar, though perhaps smaller, trend of ocean surface tem- erature, as in the north Atlantic Ocean (Brown 1953).

Farther south there are only two long temperature series on the fringe of the Antarctic one, at Grytviken 55°S and at Laurie Island 61°S, and neither of these shows any overall end during the period 1904-1950. Both stations lie in a very stormy zone of pack-ice nd icebergs, where the thermostatic properties of melting ice must have a powerful influence on surface temperatures, so perhaps we should not expect to find any trend ere so long as the sea-ice is plentiful.

(*b*) *Overall temperature trends in the zones*

The difference between succeeding 30-yr averages is a convenient way of recording ng-period temperature changes, when only the general trends are required, without he fluctuations of moving average or accumulated temperature graphs. It is not, however, eally satisfactory for mapping the trend over the earth from individual stations, because n unsuspected lack of homogeneity at an isolated station may be magnified to cover a reat area with the wrong sign. The half-period change 1891 to 1950, together with the pproximate areas covered by the stations used here, is shown in Table 1.

The overall change marked N – T – S in Table 1, is simply the average for the 3 najor zones, because it was found that a more elaborate system of weighting according o the area covered by stations in each zone, and assuming this change was zero over he Antarctic, led to virtually the same result for the whole earth.

TABLE 1. 30-YR CHANGE 1921-1950 MINUS 1891-1920 INTEGRATED FOR THE ZONES

Zone	Latitude range of zone	Zone area per cent Earth	Approx. station cover per cent zone	Average for zc °C 1921-1956 minus 1891-19
Sub-arctic	73-60 N	4½	80	0·83°
North temperate	60-25 N	23	72	0·39°
Tropical	25 N-25 S	42	75	0·17°
South temperate	25-50 S	17	25	0·14°
N – T – S	60 N-50 S	82	63	0·23°

(c) *Seasonal trends*

Although only annual temperature trends are illustrated here, a word should be sai about those for the seasons, because there appears to be a widespread impression tha it is only the winter which has advanced. Although true for certain ice-infested distric of the sub-Arctic, this is far from being the case in several major regions of the nort temperate zone, where the summer (June to August) temperature has risen more than th annual. In Table 2 the summer and annual 30-yr changes are shown for a few reliab stations, which have an annual trend quite typical of the region in which they are situate Out of 40 stations in the north temperate zone, for which the summer temperatures wer examined, only Valentia, Father Point, Tomsk and Irkutsk, showed rather less rise i summer than for the year.

TABLE 2. 30-YR CHANGE 1921-1950 MINUS 1891-1920 FOR SUMMER AND YEAR (°C)

Station	Region	Summer	Year	Station	Region	Summer	Yea
Oxford	England	0·52	0·34	Suttsu	N. Japan	0·60*	0·2
Neuchatel	Switzerland	0·55	0·41	Hikone	S. Japan	0·71	0·1
Sonnblick	High Alps	0·75	0·49	Blue Hill	NE. U.S.A.	0·72	0·6
Irkutsk	Central Siberia	0·33	0·40	Alpina	Great Lakes	0·68	0·4
Tomsk	West Siberia	0·29	0·35	Concordia	Mid-west	0·69	0·5
Tashkent	Central Asia	0·29	0·08	Winnemucca	Plateau	0·91	0·5

* July to September

5. CAUSE OF THE RISING TEMPERATURE TREND

The cause of climate fluctuations and trends is speculative in the present state c knowledge, but a few brief remarks on this subject seem called for here. These are con fined to certain radiative effects in the atmosphere, because limitation of space does no allow the discussion of several other theories which have been proposed from time t time.

(*a*) *Atmospheric dust as a cause*

It has been suggested by Wexler (1953) that the rising temperature trend could be e to increased atmospheric transparency for solar heat, owing to a reduced number dust-producing volcanic eruptions in recent decades. There is no doubt that the short- iod fluctuations in the zones (Fig. 3) lend some support to this view, for they strongly ggest disturbances to the radiation balance affecting all three zones in the same way her than the transport of heat from one zone to another. For example, the high-altitude st from Krakatoa 1883, and Pelée 1902, could have caused the sharp falls in all three es about those dates, and the absence of concurrent fluctuations since 1920 might be ributed to a lack of explosive eruptions of sufficient size to affect the whole earth. t the problem is very complex, and it would be difficult to explain the big illations about 1912-20 on the basis of the Katmai eruption in 1912, or the fact t the many eruptions since 1920 do not appear to have had a significant effect the zonal fluctuations. Evidently there are other factors at work besides volcanic st.

If the upward temperature trend shown by the zonal averages (Fig. 4) were due to increase of solar heat reaching the earth's surface, either because of greater atmospheric nsparency, as proposed by Wexler, or a rise in the energy output of the sun itself, e would expect to find a greater rising trend in sunny sub-tropic countries than in the udy north Atlantic region. But the reverse is observed, in fact some of the sub-tropic d desert regions are notable for their absence of any temperature trend in the last 60 more years – particularly sunny Australia and the Lake Balkhash-Caspian region of entral Asia. Also there would be reason to expect increased precipitation in the tropical in belt if the solar heating grew stronger. In fact a decrease has been reported in some rts of this region (Kraus 1955).

The alternative theory that a warmer and drier trend has been caused by decreased lar heating is very hypothetical. It would require a decrease in the vigour of the general rculation, with reduced polar temperatures, both of which are contrary to observation far as the North Atlantic sector is concerned (Lamb and Johnson 1959).

To summarize this brief discussion it could be said that, although a solar or dust ypothesis is applicable, or even necessary, to explain the short-period fluctuations found the zones, it fails as an explanation of the pattern of recent climatic trends observed various parts of the world.

(*b*) *Increased carbon-dioxide radiation as a cause*

Some years ago the writer suggested that rising temperature trends, already observed certain regions, could be due to back radiation from the extra CO_2 produced by fossil- uel combustion. (Callendar 1938, 1949). Since then calculations on atmospheric radia- ion by Plass* (1953) have supported this view, and he considers that variations of atmos- heric carbon dioxide are an important factor in climatic change (Plass 1956). Here it ill be assumed that the essential features of this hypothesis are well known, and also hat a rise in the amount of atmospheric carbon dioxide has actually occurred since the urn of the century* (Callendar 1958). But if these factors are taken for granted an attempt

* For very small changes of CO_2 in the atmosphere Plass finds a surface temperature derivative, $dt/d\ CO_2$, of 12, in °C and m cm^{-2}. From this a rise of 0·4° (North temperate zone, Table 1) would require 33 mg cm^{-2} more CO_2 in the atmosphere, equal an average increase of 7 per cent between 1891-1920 and 1921-1950. This compares with an observed increase of CO_2 over these eriods of approximately 6 per cent in the European region, (Callendar 1958). It should be remembered, however, that this matter f atmospheric CO_2 increase is highly controversial at the present time, and several authors (Revell and Suess 1957; Flohn 1958, nd others) have expressed doubt as to the possibility of a CO_2 increase approaching the amount, (40 mg cm^{-2} by 1950) added by ossil-fuel combustion.

must be made to show that the observed distribution of recent climatic trends over t earth is not incompatible with the CO_2 hypothesis, and that in certain cases the latt can supply a reasonable explanation. It must be emphasized, however, that this hypothe can only explain general trends, for the short-dated fluctuations are clearly due to qui other factors.

There are many features of the recent climatic fluctuations and trends which cann be explained in the present state of knowledge, but it is possible that the CO_2 hypothe can supply the answer to more than one of them. Examples of this explanati follow :

(1). It is observed that the rising temperature trend is very small in the south tem perate zone as compared with that in the north (Table 1). Apart from the natural tim lag due to the thermal inertia of the great southern oceans, with their constant supply melting icebergs, there is also a large, but unknown, time lag in the transport of exce CO_2 from where it is nearly all produced, in middle northern latitudes, across the equat into the southern westerlies. Thus a considerable fraction of the extra CO_2, and th warming effect which goes with it, may still remain in the northern westerly circulatio to give the greater temperature rise there than in other latitudes. This argument appea to be supported by radio-carbon measurements showing a much smaller proportion c fossil carbon (i.e., from coal and oil) in living vegetation taken from the Southern Hemis phere than in samples from the north temperate zone (Arnold and Anderson 1957).

(2). The tendency for precipitation to decrease in many warm regions, and remai sub-normal during the first three or four decades of this century (Kraus 1955). In thi case the following explanation, although plausible in a general sense, does not presume t cover the details of this fluctuation in either time or space. As the quantity of precipitatio is a measure of the latent heat released at the cloud level, anything which retards th loss of this heat will tend to slow down the convection currents and precipitation. Increase CO_2 in the atmosphere would be effective in reducing the loss of heat by radiation fro the upper surface of clouds, the more so because there is seldom enough water vapou above the cloud level to interfere with radiation in the CO_2 wave bands. Thus mor CO_2 should tend to depress the convective precipitation in the tropics, at least for a tim while temperatures rise to restore the overall balance between heat supply and loss. I this connexion it has been pointed out by Kraus (1956) that the rainfall has apparentl not decreased in the area of the SE Asia monsoon, where land heating drives the system Therefore it may be that the thermal inertia of the oceans has delayed the re-establishmen of radiative equilibrium in other tropical regions, with consequences as suggested.

(*c*) *Trends in the Arctic region*

There is no doubt that the most spectacular climatic event, of which we have reliabl instrumental record, has been the big rise of temperature in most parts of the sub-arcti zone during the 1920's and 1930's (Fig. 3). This rise, however, has been quite local i relation to the major zones, and it cannot have a direct connexion with variations o atmospheric radiation, whether caused by CO_2 or any other factor. On the other hand an indirect connexion is quite feasible because warming in lower latitudes could cause th polar vortex to contract and bring the westerly circulation, with its storms and clouds more often into this marginal zone.

The significance of the latest developments in the sub-arctic zone, which include a marked temperature recession in some sectors, 1948-1956, cannot be judged as yet, bu there is little sign of this recession spreading to temperate latitudes, where 1957-195 have been warm years in many parts.

Acknowledgments

I should like to take this opportunity to thank Dr. D. J. Schove for the loan of much luable temperature data, and also to thank the staff at the Meteorological Office Library r untiring assistance in procuring obscure publications from the catacombs of that stitution.

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World Weather Records (1941-1950)	1959	*Smiths. Misc. Coll.*, **79**, **90** and **105**, U.S.W.B., Washington.

Appendix

Annual deviations integrated for the zones from mean 1901-30, °C × 100

Sub-arctic zone. 60-73°N

Decade	No. of stations	1	2	3	4	5	6	7	8	9	10	Decade mean
1870	8				− 50	− 27	− 49	− 63	26	− 16	− 60	− 34
1880	14	− 100	− 135	− 8	− 57	− 85	− 43	− 73	− 110	− 4	− 35	− 65
1890	18	− 70	− 102	− 62	6	− 37	− 18	− 2	− 47	− 58	− 31	− 42
1900	20	4	− 127	13	− 5	32	− 14	− 30	0	− 18	− 38	− 18
1910	23	− 25	− 9	7	0	− 4	9	− 62	− 59	− 37	26	− 16
1920	29	17	− 2	42	47	2	77	− 10	95	52	47	36
1930	46	57	50	− 12	107	37	55	110	132	87	47	67
1940	59	30	55	129	119	49	41	94	86	31	29	66
1950	34*	0	40	100	90	− 30	0					

(N) North temperate zone 25-60°N

1870	60	− 25	4	11	− 22	− 66	− 25	− 11	32	− 22	− 9	− 15
1880	136	− 11	10	− 40	− 48	− 35	− 24	− 19	− 42	− 3	− 3	− 21
1890	159	− 22	− 28	− 34	− 3	− 26	− 21	− 11	8	11	23	− 10
1900	169	13	− 13	− 19	− 14	− 8	12	− 35	− 18	− 14	5	− 9
1910	171	− 6	− 35	0	16	15	− 10	− 36	+ 10	− 8	− 1	− 7
1920	187	39	10	20	1	41	25	12	9	− 24	28	16
1930	159	31	42	− 8	54	16	34	22	65	55	24	34
1940	200	22	18	51	53	13	56	36	60	53	27	39
1950	100*	34	35	74	36	44	10	40				

(T) Tropical zone 25°N-25°S

1870	10	− 2	2	9	− 11	− 6	− 8	26	56	− 19	2	+ 5
1880	30	13	− 20	− 20	− 32	− 9	− 13	− 43	11	20	− 18	− 9
1890	44	− 1	− 5	− 44	− 19	− 4	15	30	− 8	9	30	+ 1
1900	63	10	23	− 6	− 35	1	6	− 12	− 10	− 13	− 28	− 7
1910	75	− 10	3	− 8	21	41	− 10	− 43	− 22	14	1	1
1920	91	− 2	4	0	4	− 7	23	13	27	8	12	8
1930	94	28	15	0	− 3	4	16	17	10	8	26	12
1940	151	48	31	− 2	13	13	25	30	26	18	4	21

(S) South temperate zone 25-60°S

1870	11	− 34	− 1	2	− 32	− 13	15	15	− 24	− 31	− 4	− 10
1880	13	− 10	− 15	− 24	− 46	− 25	− 12	− 23	− 7	− 19	− 16	− 20
1890	17	− 20	− 30	− 23	− 14	− 5	37	7	− 37	1	26	− 6
1900	37	13	− 15	− 20	− 10	− 28	− 20	− 21	− 4	− 5	− 4	− 11
1910	38	− 10	0	22	20	7	12	22	− 2	30	6	11
1920	36	− 10	12	− 6	− 6	− 5	17	1	5	7	− 8	1
1930	39	1	19	7	13	− 3	8	10	9	2	27	9
1940	46	11	10	25	32	12	13	31	6	3	5	15
1950												

N − T − S (North-tropical-south) 60°N-60°S

1870	80	− 20	5	0	− 22	− 28	− 6	10	21	− 24	− 4	− 7
1880	159	− 2	− 8	− 28	− 41	− 23	− 16	− 28	− 12	0	− 12	− 17
1890	220	− 14	− 21	− 33	− 12	− 11	11	9	− 12	7	27	− 5
1900	269	12	− 2	− 15	− 20	− 12	− 1	− 23	− 11	− 11	− 9	− 9
1910	284	− 9	− 11	5	19	21	− 3	− 19	− 5	12	2	1
1920	314	9	9	5	0	10	22	9	14	− 3	11	9
1930	292	20	25	0	21	6	19	16	28	22	26	18
1940	400	27	20	25	33	13	31	33	31	25	12	25
1950												

(* 1951-1957 without U.S.S.R. or about one-third of the zonal area)

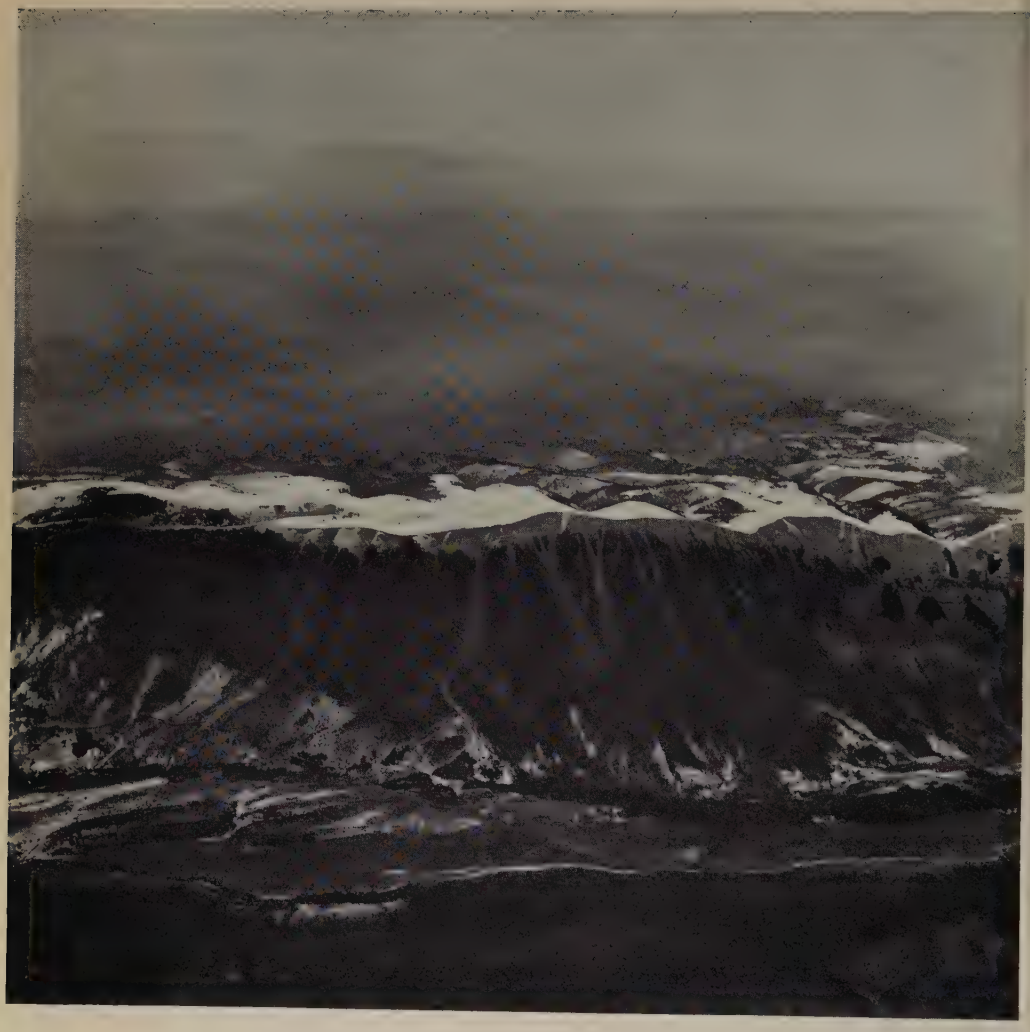

Figure 1. Montagne de Lure viewed from the north, with the ridge of Pelegrine in the foreground.

551.558.21

Experimental studies of lee waves in the French Alps

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Read before the Society by N. Gerbier 20 Jan. 1960; communicated by R. S. Scorer

1. Brief note on observational methods

Observations have been made during 1956-1959 on atmospheric waves in the French ps, through the cooperation of Le Météorologie Nationale, the Service de la Formation ronautique du Travail Aérien et des Transports, and the Comité d'Action Scientifique la Défense Nationale.

The National Gliding Centre at St. Auban sur Durance (Fig. 2) was chosen as the se for the field work because it was particularly well suited for the study of waves. is 450 m a.s.l. at the confluence of the rivers Bleone and Durance. 12 km to the north- st is the Montagne de Lure (Plate I, Fig. 1) (1,827 m a.s.l.) which is oriented east to st, and stands about 1,400 m above the surrounding countryside. There are other rallel ranges further north which are 500-600 m lower. To the south-east the ground fairly flat for about 40 km on the plateaux of Puimichal and Valensole (around 800 m s.l.).

It was well known that waves occurred in the lee of Mgne. de Lure in a N or NW'ly rstream, and significant waves are also produced by other mountains with other wind rections. In particular we may mention the Cheval Blanc (2,314 m a.s.l.), 35 km east- rth-east of St. Auban and oriented north to south, and the range of Dévoluy near the l of Lus-la-Croix-Haute, which is 70 km to the north-north-west of St. Auban.

The chief items of equipment assembled for the research are now described :

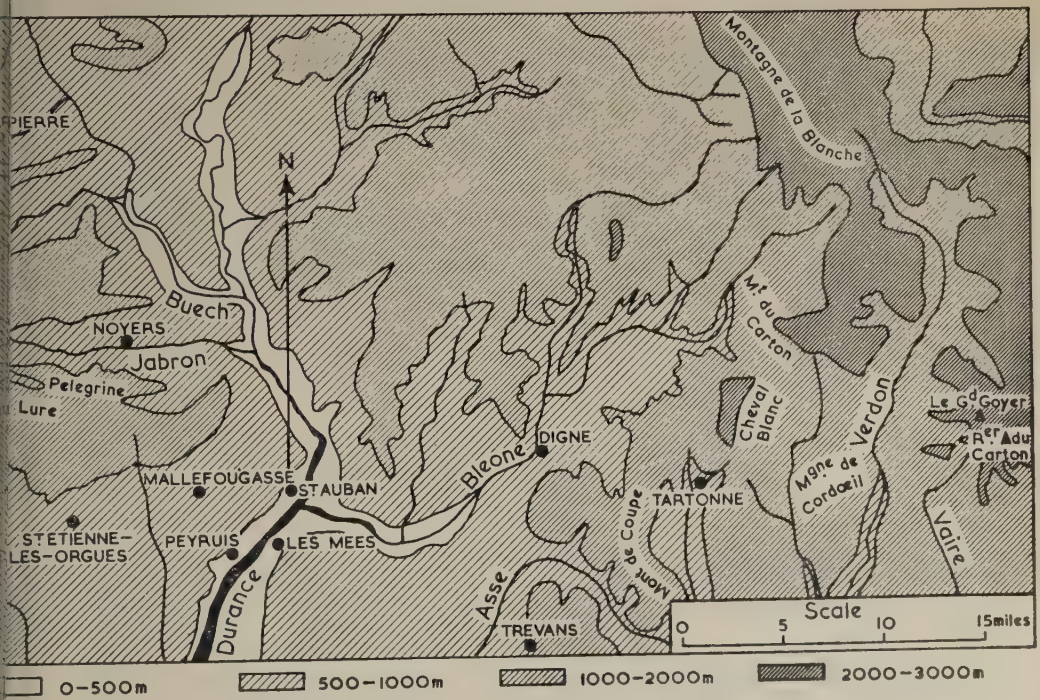


Figure 2. The immediate neighbourhood of St. Auban.

(i) *Radiosonde*

A radiosonde station with a radio theodolite was installed at Mallefougasse (see map Fig. 2) at 730 m on the southern slopes of Mgne. de Lure. Balloons released in a NNW'l wind thus travel through any waves that may exist.

At least one ascent was made each day during the three series of observations in the afternoon for the first series, and around 08 GMT for the other two series. On any day of particular interest two ascents were made. Zero-lift balloons were also sent up from this station from time to time.

(ii) *Radar*

An automatic 'Cotal'-type radar was installed on the south-east boundary of the area. It was used to follow the radiosonde balloons, free balloons released from the ground to give additional wind measurements, and zero-lift balloons.

The accuracy obtained is quoted as $\pm$ 15 m for oblique distance and 0°·03 for elevation (while in motion).

(iii) *Gliders*

We had available at first two, and later, three, gliders; two were of the Breguet 901 single-seater type, and one was a Bregnet 904 two-seater type. The pilots were under the direction of Monsieur J. Landi, chief pilot of the experimental section at St. Auban.

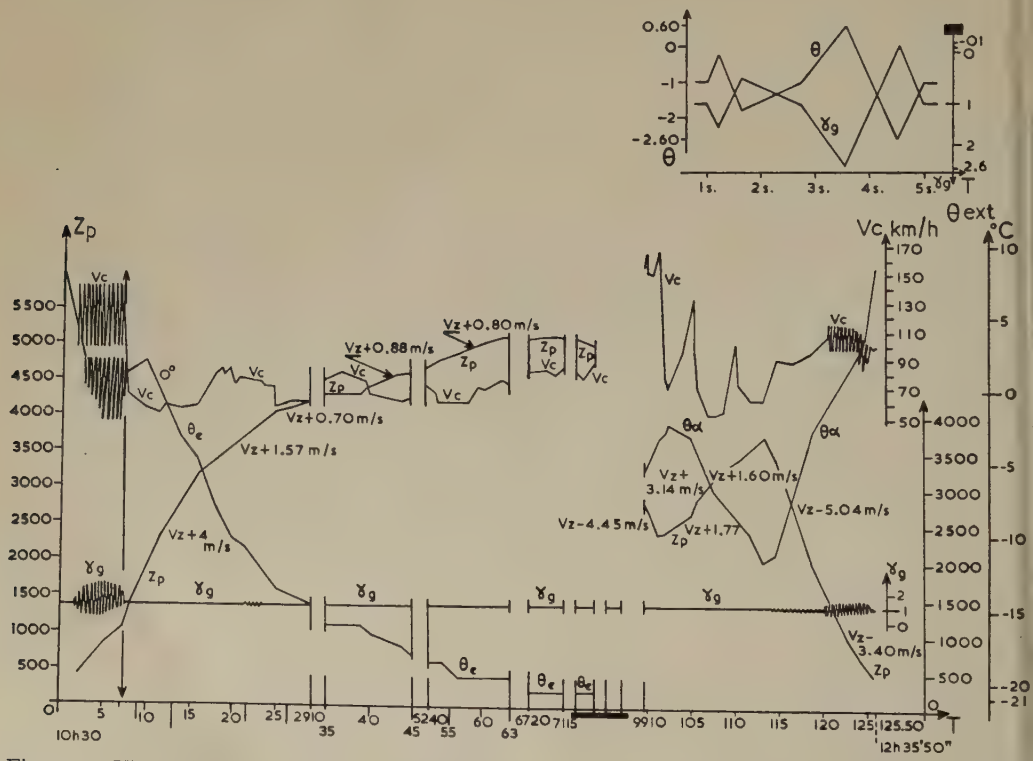
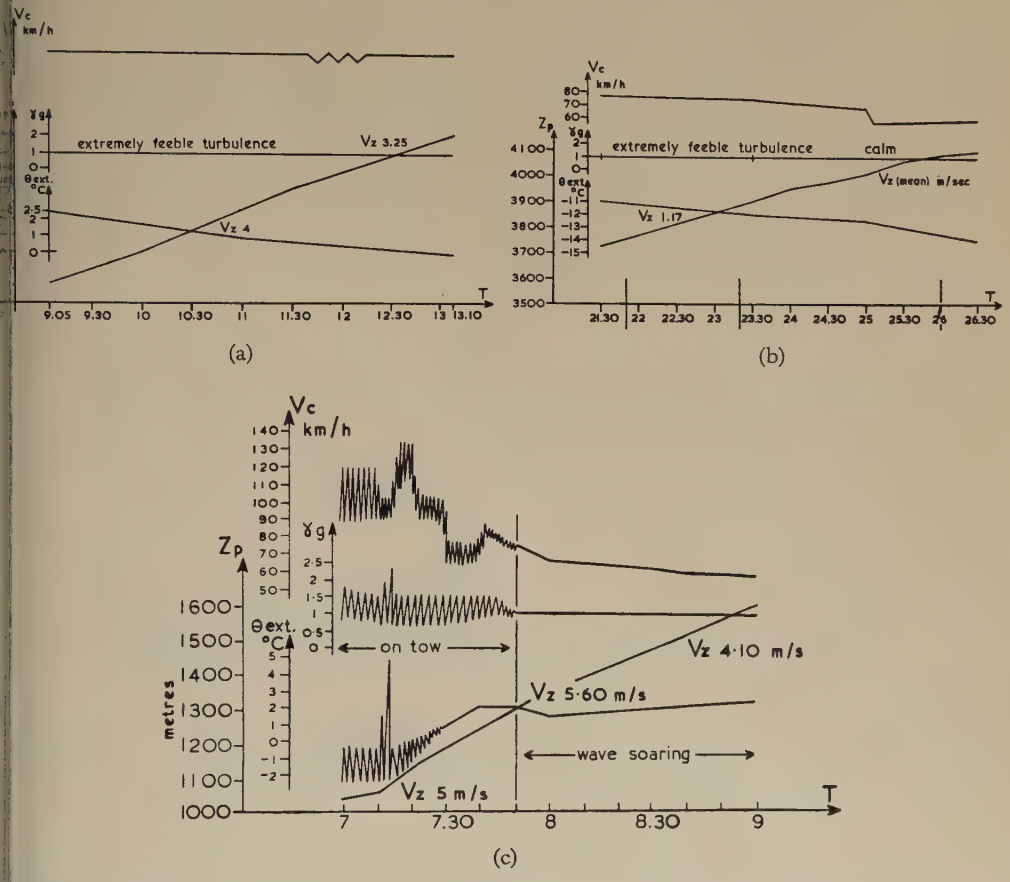


Figure 3. The analysed record of the A.20 apparatus, for a glider flight on 25 Jan. 1956. Time is measured horizontally in minutes and seconds from take-off. The tow-plane was released 7 min 25 sec after take-off. The zig-zag traces represent rapid fluctuations of the magnitude indicated. Several cuts have been made where the record is not of interest. The inset diagram is an enlargement with a time scale in seconds, showing the nature of the record (actual points) and the correlation between temperature θ_c and acceleration γ_g in a turbulent region. Other quantities recorded are Z_p (pressure altitude), V_z (vertical velocity) which is marked in at several points on the altitude record, and V_c (corrected, i.e., true, air speed). The vertical velocity is obtained from the slope of Z_p.

The gliders were equipped with oxygen and carried a meteorograph recording ssure, outside temperature and humidity, continuously. They also carried an A.20 paratus recording as a function of time, pressure altitude, stagnation temperature, gential and normal accelerations, and airspeed. With these several readings the h of the glider could be followed in detail. An example is given in Fig. 3.



gures 3 (a) and (b) show extracts from the record with the time scale enlarged, made when the glider was ring over a fixed point on the ground. Fig. 3 (c) shows an enlargement of the record at the moment of entering the smooth wave lift when the tow plane was released.

(iv) *Zero-lift balloons*

These were 700 g rubber balloons enclosed in a silk envelope 260 cm in diameter hich could withstand a pressure of 50 g cm^{-2} without appreciable stretching. Each lloon was inflated so as to occupy exactly the envelope at a predetermined pressure. carried a parachute, radiosonde transmitter, radar reflector, and a small weight to give the required total weight.

Each balloon was carried to the desired pressure height by a tractor balloon, attached single nylon thread which was cut by an electric device when the desired equilibrium ight was reached.

In each series of observations about seven balloons were thus launched for each of e pressure heights 700 mb, 500 mb and 250 mb. The release point was chosen so that e equilibrium height should be reached at the beginning of a line of waves. Examples balloon tracks in relation to the topography are shown in Figs. 4, 5, 6.

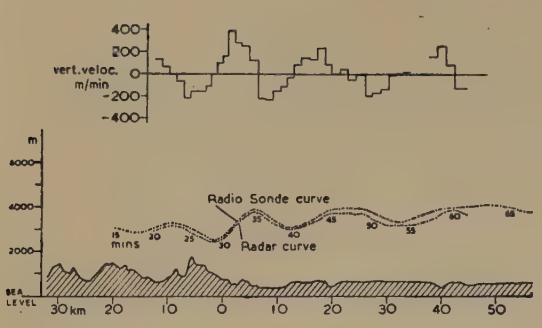


Figure 4. Track of a zero-lift balloon in relation to the topography along the line Orpierre-Mallefougasse, 25 Jan. 1956. Positions are shown every minute as obtained by the radar and by means of the radiosonde. The mean vertical velocities for one-minute intervals are shown above.

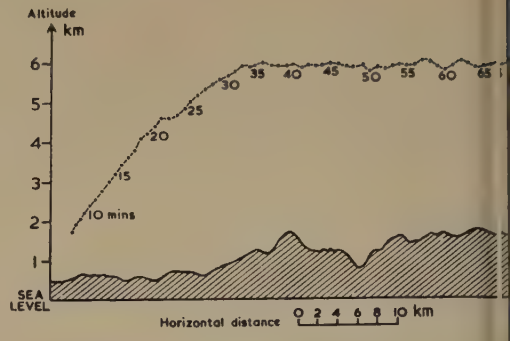


Figure 5. The track of a zero-lift balloon on 3 Fe 1957. The points give the position every minut The wave amplitude increases over the second an third mountains although it is small in the le of the first.

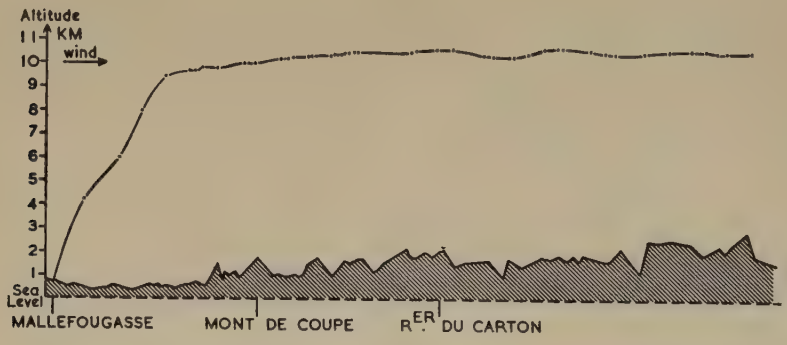


Figure 6. The track of a zero-lift balloon released from Mallefougasse towards the east on 5 Feb. 195 The points plotted are selected to give the shape of the curve. The topography is very complicate in this region.

(v) *Powered aircraft*

In the last two series of observations powered aircraft based on Istres were used High-altitude turbulence in particular was investigated but only a few observations hav so far been made and the methods need further development.

2. The nature of the conclusions drawn

Observations were made on waves set off by Mgne. de Lure in NNW'ly airstreams by Cheval Blanc in W'ly airstreams, and by Mgne. de Lure and Mt. Ventoux (a west ward continuation of the Lure range) in SE'ly and ENE'ly winds. In certain case waves due to two mountain ranges have been simultaneously studied.

Although certain aspects, in particular rotors and high altitude turbulence, nee more detailed study than has so far been possible, some conclusions can be drawn abou the influence of the invariant factor, relief, and varying factors, wind and airmass, o wave formations. The details of all the observations are not given here : they ar available in a report by Gerbier and Berenger (*Monographie de la Météorologie National* Aug. 1960).

(*i*) *The invariant factor – relief*

Clearly, without some relief there would be no lee waves. For the same wind profile and airmass, different mountains produce different wave systems, and the evident effects may be summarized as follows :

(a) An isolated peak or hill has less influence than a mountain range or large mountain mass.

(b) For the same wind, the region in which eddying motion is found, is larger and the eddies more pronounced in the lee of a steeper slope.

(c) If the line of the ridge crest is concave towards the wind, the wave effects are more pronounced than for a straight crest. For a crest line convex towards the wind the effects are less pronounced.

If the crest line varies, the greatest lift is produced in the neighbourhood of the parts concave towards the wind.

(d) For the same aerological conditions, higher mountains produce more developed waves.

(e) Each successive mountain produces its own effect which complicates the wave pattern due to mountains further downstream. There is sometimes a damping of the lee waves if the spacing between mountain tops and the wave lengths are out of phase, or an amplification if they are in phase.

(*ii*) *Variable factor – wind*

(a) By far the most pronounced effects occur when the wind blows from a direction within 30° of the perpendicular to the crest line. It is obvious that in the case of a long ridge there is no effect when the wind blows along it.

(b) The effects are greatest when there is little or no change in direction of wind with height. If the wind is parallel to a range at a certain level, the waves usually appear to be confined below that level. As we shall see later, the effect is as though there were a discontinuity in the airstream at that level.

(c) If the wind direction changes with time so as to become parallel to the ridge at any level, wave lift usually disappears above that level, and lee waves disappear altogether if this happens at the level of the mountain crest.

(d) It seems that the wind speed must exceed a certain minimum before lee waves appear, otherwise lift can only be found near the top of the hill and this dies away rapidly with height. In our experience, if the wind is less than 10 m sec^{-1} at the hill tops no lee waves are detectable.

(*iii*) *Variable factor – airmass (lapse rate)*

Very often there is not a single airmass but two or three airmasses superimposed up the tropopause. What will be their effect on lee-waves ? All other factors remaining constant, our observations indicate that where there is an inversion of temperature or an isothermal layer, the vertical amplitude of the waves is a maximum.

However, if two different streams are superimposed (for example, an increase in speed in a warm upper layer), it is possible that forward-moving waves, sometimes with rotors, will form at the discontinuity. These progressive waves will periodically come into phase with the lee waves and produce an increase in vertical amplitude, which is preceded and followed by a decrease. By this hypothesis we can explain certain variations in wave-cloud thickness which occur in a fairly short time. In the majority of cases, however, variations in cloud structure are due to variations in humidity.

3. Some speculations on the modification of lee waves through variations of the flow in space and time

Most of the time the waves in the lee of a mountain chain do not behave accordi to the simple schematic pattern shown in Fig. 7. In reality there most often exist mo than one wave train with different wavelengths and amplitudes because the wind ar airmass characteristics vary with altitude. It is helpful therefore to study a number wind profiles and some of the time and space variations in the lee-wave patterns whic go with them.

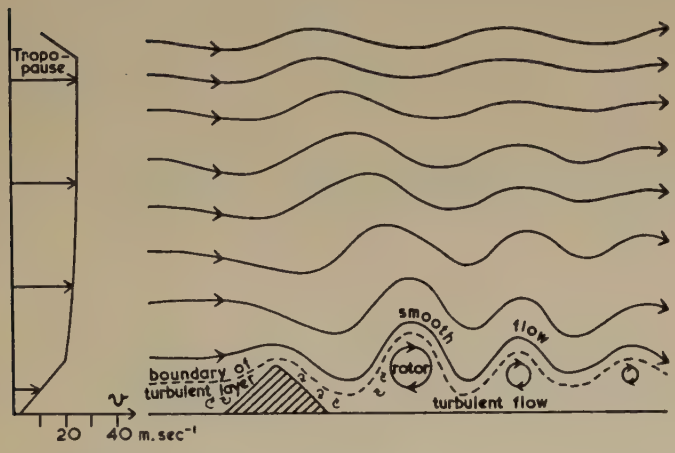


Figure 7. This diagram is schematic and is based on observations. It shows the wave pattern in its simplest form. With this kind of wind profile the region of lift slopes towards the mountain from the first lee wave but lift in the second lee wave may be vertical or even sloped away from the mountain if the wind increase is substantially greater than here indicated (also see Fig. 11).

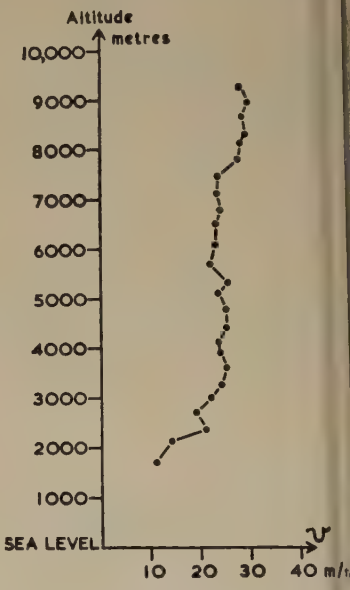


Figure 8. The wind profile o 26 Jan. 1957.

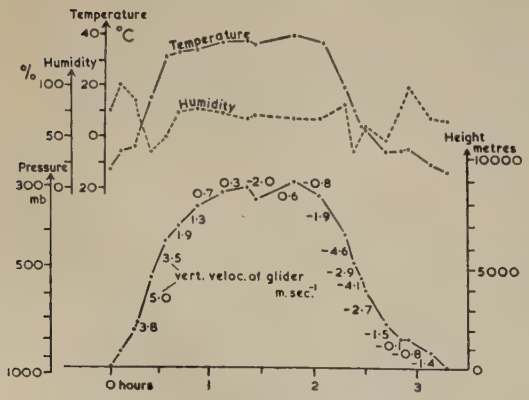


Figure 9. Glider meteorograph record for a wave flight on 26 Jan. 1957. The mean vertical velocity of the glider is indicated against the height curve in m sec^{-1}. The sinking speed of the glider in still air is about 1 m sec^{-1}.

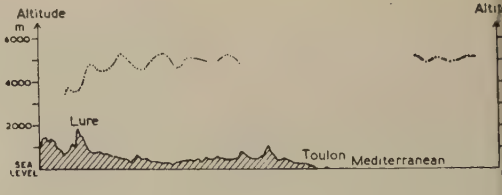
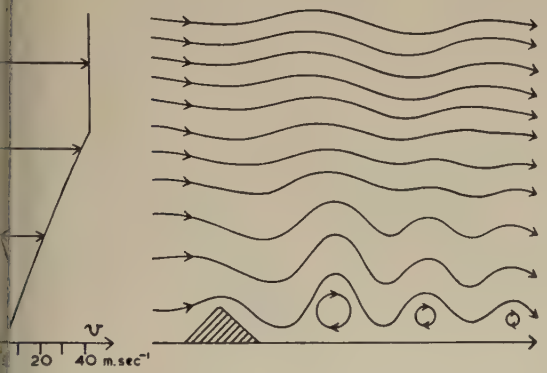


Figure 10. Track of a zero-lift balloon 26 Ja 1957, released at 1400 GMT. The balloon w abandoned while a second one was prepared f release but was picked up again when the secon balloon burst. Waves can be seen far out ov the Mediterranean.

(i) *The dependence of lee waves on the profile of wind component normal to the crest line in a stable atmosphere*

(a) Wind effectively constant : up to the tropopause such a simple profile produces flow similar to the scheme shown in Fig. 7. This is illustrated by the case of 26 Jan. 957 where wind profile is shown in Fig. 8. On this day, in a NNW wind, gliders reached ights exceeding 9,000 m (max. 9,380 m) in the lee of Mgne. de Lure. The meteoro- aph record of a glider is shown in Fig. 9. The track of a zero-lift balloon which shows e waves well is given in Fig. 10.

(b) Substantial increase of wind with altitude : usually a progressive increase in avelength with altitude is observed, as already noted in Fig. 7. The flow appears to llow the schematic pattern shown in Fig. 11, and the velocity profile is exemplified y the case of 5 Feb. 1958, shown in Fig. 12. Because of the diminution of vertical ampli- de the gliders reached their ceiling at 6,000 m in the lee of the Cheval Blanc massif.



igure 11. Schematic flow pattern with wind and wave- length increasing with height.

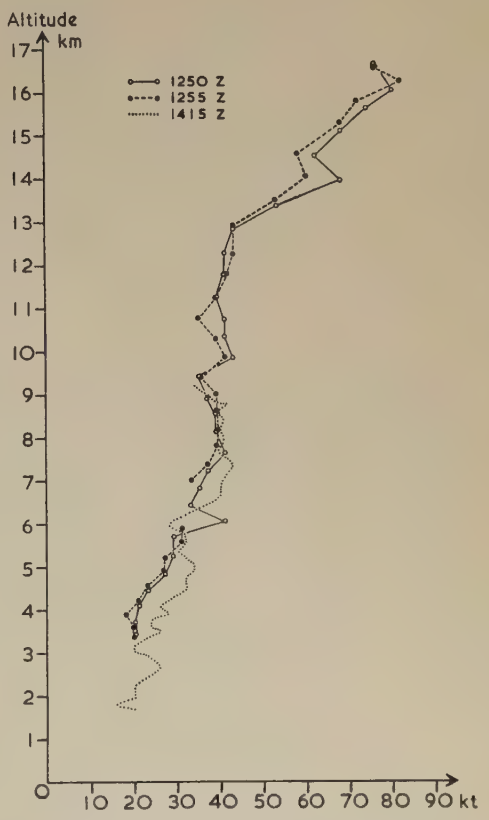


Figure 12. Radar wind at Mallefougasse, 5 Feb. 1958. Three successive ascents are shown.

(c) Sharp increase in speed in a shallow layer : there appear to exist two wave trains of different wavelength separated by a turbulent layer situated where there is large shear, as indicated in Fig. 13.

The situation of 25 Jan. 1956 for which the wind profile is given in Fig. 14 is of this type. In the course of a flight made on this day in the lee of Mgne. de Lure, the pilot, Monsieur J. Landi, recorded several levels at which there was slight turbulence. Of these, that of maximum amplitude was at 5,300 m, that is, just below the layer of greatest shear (Some details of the A.20 record of this flight are given in Fig. 3).

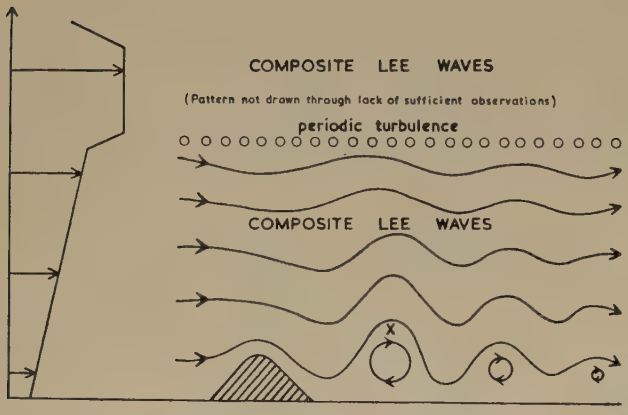

COMPOSITE LEE WAVES
(Pattern not drawn through lack of sufficient observations)
periodic turbulence
COMPOSITE LEE WAVES

Figure 13. Schematic flow pattern when there is a shallow layer of large shear.

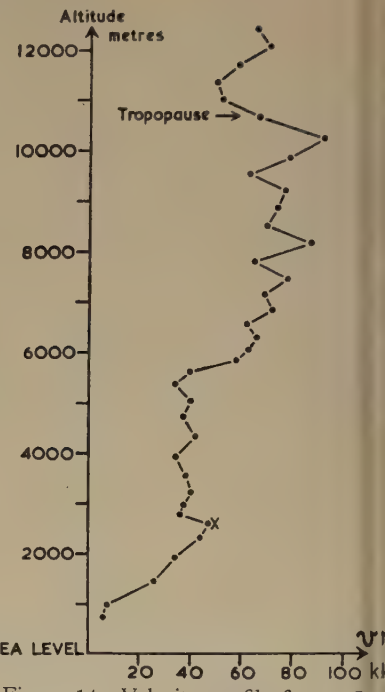


Figure 14. Velocity profile for 25 Jan 1956 08 GMT, Mallefougasse. The large velocity at about 2500 m is unrepresentative : the balloon was passing over the first rotor (position x in Fig. 13).

(d) Wind decreasing rapidly with height from a certain level : this produces a layer in which there is turbulence and which is an upper limit to the wave motion. Powered aircraft flights indicated no wave motion above the turbulent layer. In certain cases the presence of unsteady waves of much smaller wavelength may be detected, and sometimes these contain small rotors (Fig. 15).

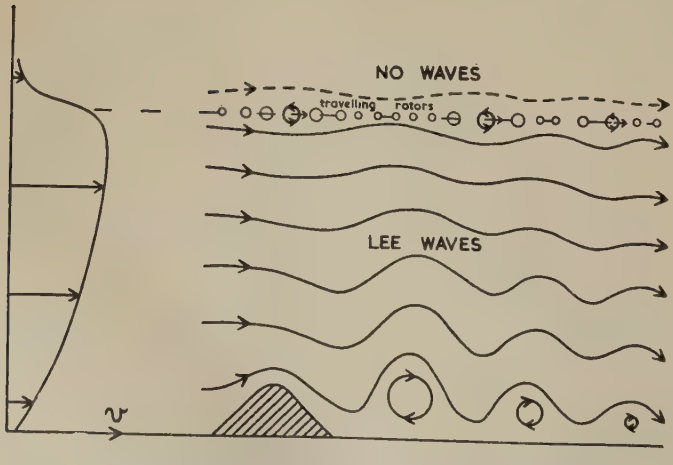


Figure 15. Schematic flow pattern when there is a sharp decrease in wind with height. In the shear layer there are often small travelling rotors.

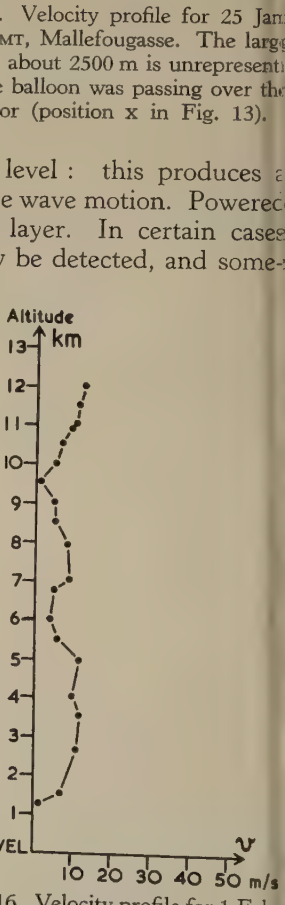


Figure 16. Velocity profile for 1 Feb. 1957. This has a sharp decrease in wind between 5,000 and 6,000 m.

The situation of 1 Feb. 1957 is of this type. The velocity profile is given in Fig. 16 has two points of interest. Firstly there was insufficient wind over Mgne. de Lure 827 m) while waves were set off in the lee of Cheval Blanc (2,300 m) and secondly re was a marked falling off of wind between 5,000 and 6,000 m.

It was necessary to aero-tow the gliders to Cheval Blanc, and to the lee of other fficiently high and suitably oriented mountains. The maximum altitude attained was 00 m. The flight track is shown in vertical section in Fig. 17.

There was no visible manifestation of waves downwind from Mgne. de Lure.

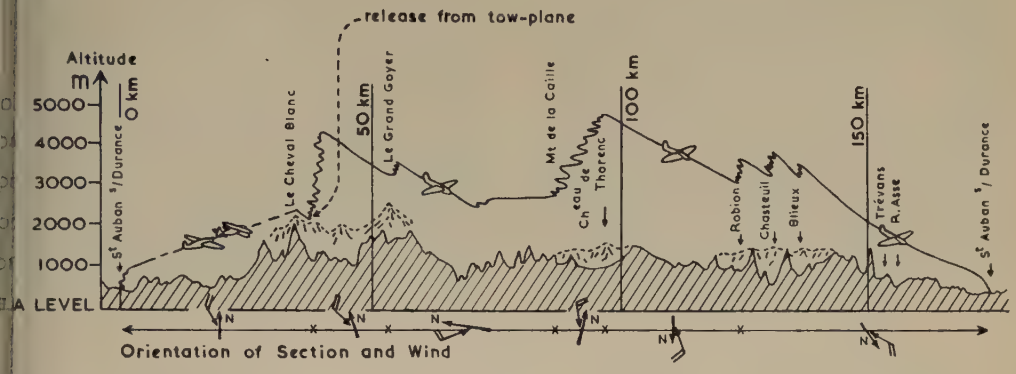
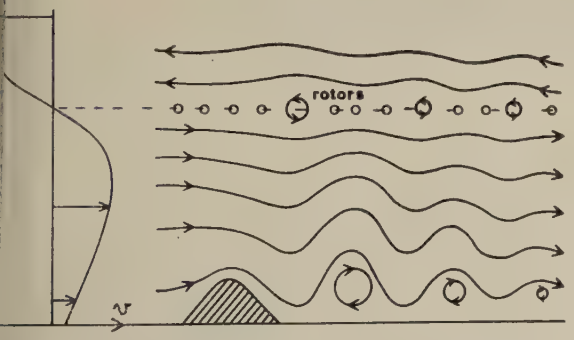


Figure 17. Flight track for 1 Feb. 1957. Between Le Grand Goyer and Trévans the track is beyond the limit of Fig. 2.

(e) Wind changing direction rapidly with height in a shallow layer : for example, change from 120° to 180° while the speed observed remained the same. In this case e have two different types of flow separated by an important turbulent layer in which ere are rotors turning in the opposite direction to those beneath the waves. A vertical ction of the flow is seen in Fig. 18. On one occasion the pilot, maintaining a fixed bsition in space, experienced a periodic variation in lift every 45 sec.

Such situations occur in troughs of low pressure, when the trough line is displaced ith altitude. A situation of this kind is that of 4 Dec. 1959. The trough at the rface had its axis at 00 GMT along the line Brussels-Toulon, but at 500 mb was on he line London-Perpignan. The profiles of wind velocity and direction are indicated in ig. 19.



igure 18. Schematic flow pattern when there is a reversal f wind direction. The rotors, or billows, are unsteady or moving.

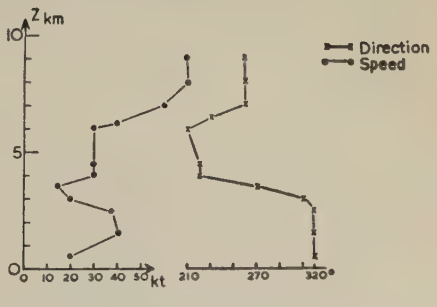


Figure 19. Profiles of wind speed and direction for 4 Dec. 1959.

The various reports on this day show a smooth ascent in waves as far as 4,000 than a layer of alternate ascent and descent with intense periodic turbulence of lar amplitude up to 5,700 m, and then again a smooth layer up to 6,000 m, at which heig the gliders reached their ceiling (see barogram, Fig. 20). The pilot, Monsieur G. Roussel wrote in his flight report : " Clouds of the fractocumulus type formed upwind of o position and moved relatively slowly in the wind direction (above 4,000 m). When the clouds arrived near to our position and slightly above it, our variometer showed a decrea in positive vertical velocity. At the moment when the glider was under the cloud, t turbulence increased and then diminished very rapidly up to the moment when we passe to the other side (the upwind side), while the variometer showed an increase in upwa vertical velocities. The cycle was reproduced several times. At the maximum altitud we came into a smooth regime."

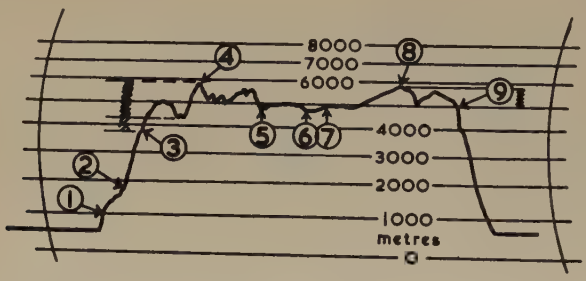


Figure 20. Barogram of the glider flight by M. D. Snrech, 1430-1700 GMT, 4 Dec. 1959.

(1) release from tow plane at Les Mées at 850 m QNH.
(2) near Peyruis, followed by ascent in lee wave,
(3)-(4) periodic turbulence of large amplitude,
(5) over Peyruis,
(6)-(8) smooth wave lift,
(7) over St. Etienne-les-Orgues,
(8) over Mallefougasse,
(8)-(9) turbulent.

Plate II, Fig. 21 (a), (b) and (c) show near the top left edge, the evolution of these fractocumulus. (a) and (c) are separated by 4 min. We were able, with the aid of a cine camera, to film these cloud sequences, and these clouds can clearly be seen turning in the opposite direction to the rotors in the lower layers.

(ii) *Effect of diurnal variations of lapse rate*

If there is an adiabatic layer of sufficient thickness the wave amplitude becomes very small, and one can expect to observe amplitude variations in lee waves when the thickness of the adiabatic layer changes. This has been observed to occur in numerous cases of NW winds. We may cite in particular, 10 Feb. 1957, when the maximum rates of ascent were made one hour before sunset after a day of convection.

Thus, the diurnal and seasonal variations of lapse rate are of prime importance; the interface between the air masses (discontinuity of lapse rate) undergoing changes of altitude in the course of the day.

(iii) *Some complex interactions*

Since the velocity profiles were calculated perpendicular to the line of crests, it follows that the diminution of velocity in the section considered could be due to a progressive or sharp change in wind direction. In this case the wavelength in the upper layer may be greater than that in the lower, with the lower lee waves acting as a mountain for the upper layer. We have, during these flights, observed wave clouds in the middle tropospheric layer when the wind is perpendicular to the wind of the lowest layers. These wavelike disturbances are probably not due to the mountains themselves, but rather to wave motions in

(a)

(b)

(c)

Figure 21 (a), (b), (c). A rotor cloud in the first lee wave with a transient rotor in a shallow layer of large shear above it, which appears in the top left corner of (a) and (b) but has disappeared in (c). This cloud was also studied by time-lapse photography.

Figure 23. The lower cloud is part of the ragged layer which formed in the first wave in the lee of Montagne de Lure, which is off the picture to the right. The gap between the first and second lee wave can be seen towards the left. The wind at a higher level was blowing parallel to the Montagne de Lure and the upper wave cloud seen in this picture was formed above the first lee wave, as though this wave were producing a mountain over which the upper air flowed at right-angles to the lower wind. The far end of the wave cloud is the upwind end.

upper layer over a lower air mass in wave motion playing the role of fixed topographic ef. 23 Feb. 1957 (Fig. 22) is a situation of this type. Plate III (Fig. 23) shows the ud (base 4,400 m, top 6,000 m) of the upper level wave situated in the layer where the nd is rotating and increasing.

In this case the maximum amplitude of this second train of waves must have been tically above the first lee wave downwind from the line of mountain crests.

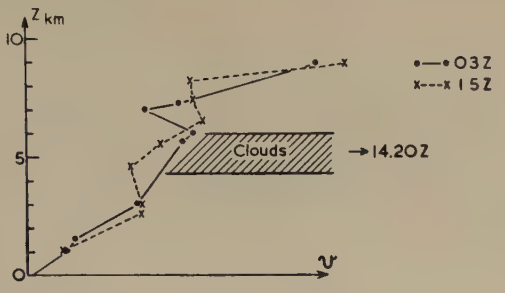


Figure 22. Wind profile 23 Feb. 1957. There was a change in direction of about 90° above 4,000 m.

We have just examined how different velocity profiles modify the shapes of waves oduced by a given relief, the airmass being presumed stable. If the winds evolve in ne, the lee-wave pattern must evolve from one type to another in accordance with it.

In the case of an approaching warm front, when the wind is increasing with time, the avelength increases accordingly and the wave clouds become more extensive, while the eaks may decrease very quickly because of the increase in humidity.

4. Conclusions

We may therefore say that wave phenomena are usually complex and we may suppose at there exist in mountainous regions three types of wave :

(1) Lee waves, accompanied by rotors when the amplitude is large.

(2) Composite waves, simultaneous existence of more than one lee-wave system with maximum amplitudes at different levels.

(3) Shear layer waves, progressive, sometimes unsteady, or with rotors or billows.

551.510.53 : 551.511.3 : 551.557

Quasi-periodic wind variations with height in the lower stratosphe

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Meteorological Office, Dunstable

(Manuscript received 24 March 1960)

SUMMARY

Quasi-periodic wind variations with height have been commonly observed in the lower stratosphe by means of the radar-sonde theodolite at Crawley. The atmospheric disturbances which give rise to the wind variations have a vertical dimension of 1 km or so, but their horizontal dimensions must be muc greater, perhaps of the order of several hundred kilometers. Perturbation theory is used to show that sld ' inertia oscillations ' with a period of the order of half a day might have such dimensions.

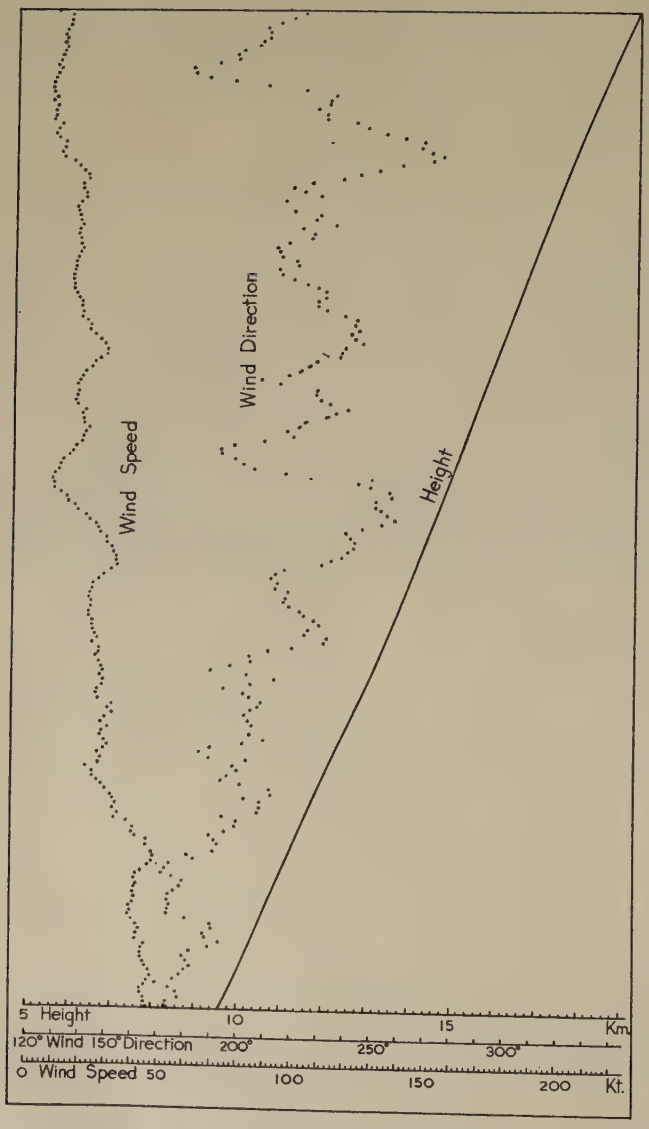


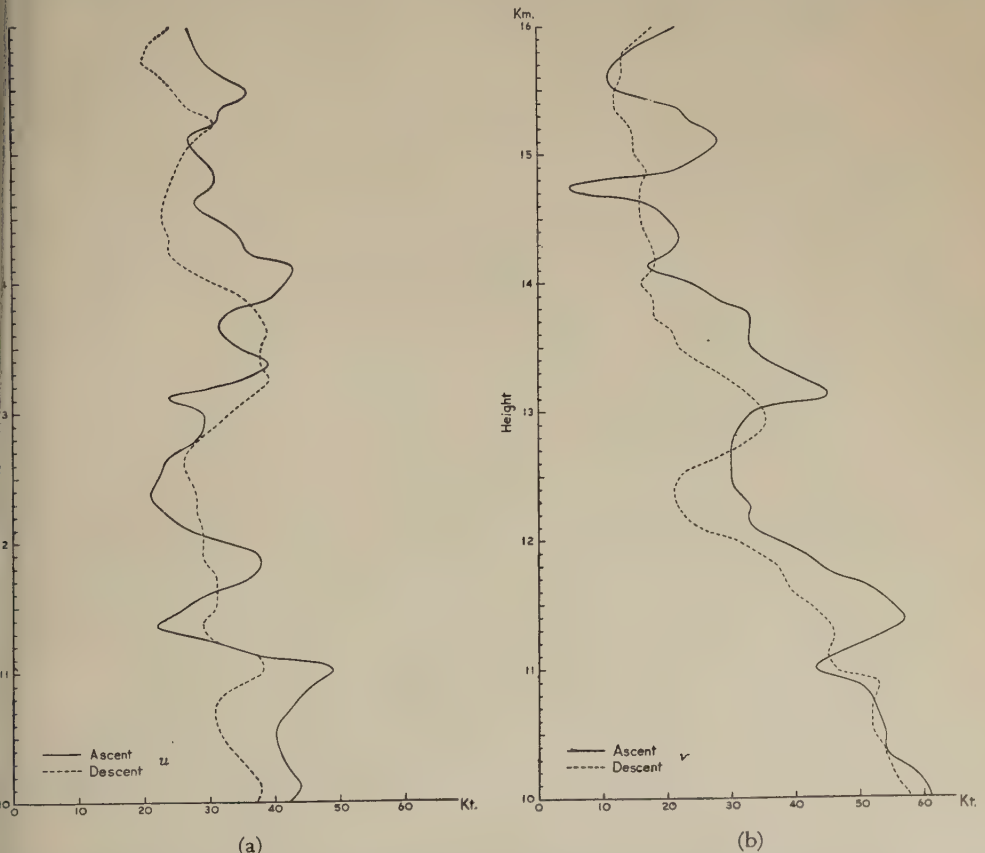
Figure 1. Part of the record of the wind determination by radarsonde at Crawley, 0500 GMT, 1 March 1959.

1. The observations

The radar-sonde theodolite was developed to give more accurate and detailed soundings he atmosphere than available from equipment in routine use. (A brief description of equipment is given by A. L. Maidens (1957)). A considerable number of soundings been made with the equipment at Crawley and the results are available in the form utomatically computed records of wind direction and speed at approximately 50 m rvals of height. A feature appearing commonly on these soundings has been the occurce of variations of wind direction and speed with height in a quasi-periodic manner in lower stratosphere.

A tracing of part of a typical record is given in Fig. 1. Although somewhat similar d variations appear in the troposphere, those in the stratosphere are usually more regular of larger amplitude. The departure of the wind from its mean through a layer of 1 or m reaches a maximum of 7 or 8 kt as the sounding balloon encounters these wind fluctuas. The variations in wind direction are most clearly marked when the winds are light are barely detectable when the winds are strong; this suggests little dependance of the plitude of the vector wind change upon wind speed.

On 14 occasions on which stratospheric wind fluctuations were noted, the Crawley nsponder was followed during its descent after the balloon had burst. Similar wind ctuations were observed during the descent of the transponder on its parachute. Most he features of the wind record during the ascent could be picked out on the record obtained ring the descent. On more than half of the soundings the agreement in regard to levels the maxima and minima of wind speed and extremes of direction was reasonably good.



gures 2 (a) and (b). Wind components 1700 GMT, 3 March 1959 during ascent and descent of radarsonde. (u – eastward, v – northward).

Fig. 2 shows the wind observations on part of the record of one ascent and on the corre ponding descent. The winds are expressed as components and the measure of agreeme is typical. Features tend to appear rather lower on the descent than the ascent and th descent record is rather smoother. This may be an effect of the inertia of the falling parachu system.

Figure 3. Northward component of velocity on five successive soundings on 24 March 1959.

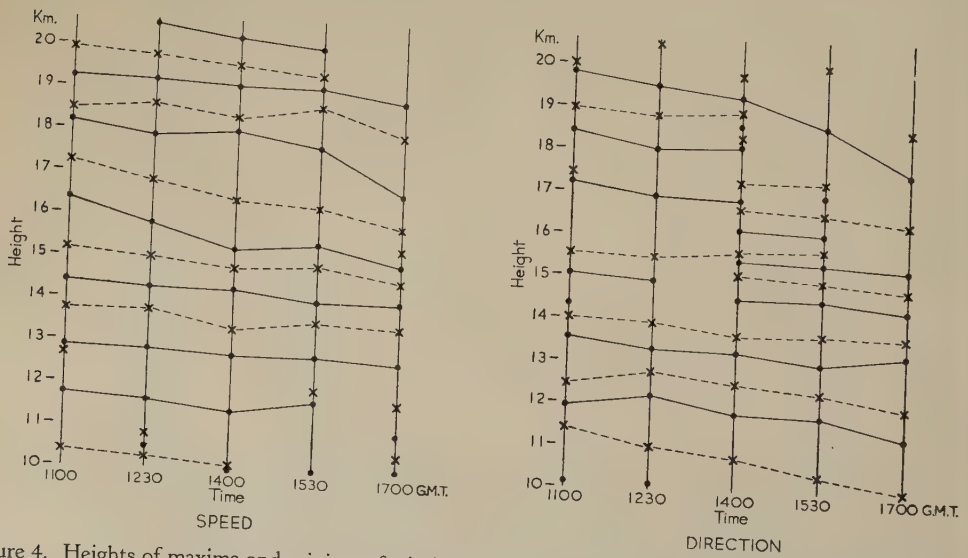


Figure 4. Heights of maxima and minima of wind velocity and extremes of wind direction on five successive soundings on 19 March 1959 (Maxima are indicated by ● and joined by continuous lines where appropriate; minima are indicated by × and joined by broken lines).

On 24 March 1959, five soundings were made spread over a period of six hours. The orthward component of velocity over the range 10 to 16 km is plotted in Fig. 3 for each scent (the velocity scale being displaced by 20 kt between ascents). Some of the irregularies of the wind appear to show a distinct persistence from one sounding to the next; in articular, the minimum of northward velocity at about 12·5 km appears on every sounding. our other sequences of five soundings at 90-min intervals are available. In three of these, me, at least, of the features of the wind profile persist through the six hours from the rst sounding to the last. Other features persist for shorter periods. This is further illustrated Fig. 4 which shows the height of the observed extremes of wind speed and wind direction s a function of time. The persistence of the features around 14 km is particularly well arked.

The hodograph for one of the series of soundings on 24 March is reproduced in Fig. 5. will be noticed that the irregular variation in the wind is about equal in the two components when the systematic decrease of the wind upward in to the stratosphere is ignored). The nd of the hodograph vector seems to describe closed loops mainly in an anti-clockwise sense. his is true of a number of such hodographs which have been plotted and suggests that the uctuations in the two components of wind tend to be out of phase.

2. Discussion

Similar fluctuations in wind with height have been reported in the observations by eans of GMD-1 equipment in America (Clem 1954). However, the wind fluctuations bserved on the American equipment are considerably greater in amplitude than those ound at Crawley, and Reiter (1958) has given strong evidence suggesting that they were nstrumental in origin.

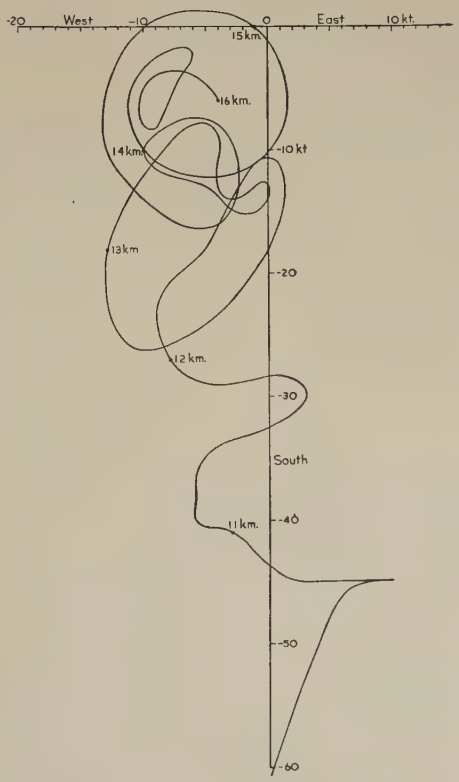


Figure 5. Hodograph of wind from sounding at 1700 GMT 24 March 1959.

On the other hand, Barbé (1958), using an automatically-following radar at Bourget-du-lac, has observed wind fluctuations more closely similar in amplitude to those found at Crawley. Moreover, Barbé has demonstrated that although these wind variations occur over layers only 1 or 2 km thick, their main features, including the altitude of maxima and minima, are reproduced on soundings separated in time by 5 to 10 min. This considerably reduces the likelihood of an instrumental cause.

Instrumental error as a cause seems unlikely at Crawley also, particularly when the agreement between observations during ascent and descent are considered. In following the descent the radar is operating at an entirely different range and elevation from that used during the ascent and it is difficult to envisage systematic errors which would be repeated under these two different sets of conditions.

Accepting the reality of these wind variations but recognising that instrumental error may be too great to give much weight to detail of wind changes within them, what can we deduce about the disturbances of the flow field which give rise to them ?

First, the disturbances are quasi-periodic in the vertical with a wavelength of the order of 1 km or so – or if not periodic, the disturbances have vertical dimensions of the order of 1 km.

Second, the horizontal dimension must be at least considerably larger than the distance between the ascent and descent of the radar transponder (say 10 to 30 km) otherwise the features of the wind field would not be reproduced. Even if the wind irregularities were travelling in the air stream there is commonly a significant shear with height, and the balloon cannot follow the horizontal motion of the air at any specific level.

Third, the duration of the wind maxima and minima seems to be at least of the order of six hours. If the disturbances were stationary, controlled by orography perhaps, then the wind maxima might remain at the same levels indefinitely. However, the hills in the neighbourhood of Crawley do not exceed a few hundred feet and it seems unlikely that the disturbances are stationary orographic features. If, in contrast we assume that the disturbances are moving with the wind (at say 10 m sec^{-1}) then the horizontal dimensions of the disturbances must be at least 400 km.

Fourth, we note that the wind irregularities involve wind shears in the vertical of the order of 20 m sec^{-1} km^{-1}. If the winds were geostrophic the thermal wind relation would require horizontal temperature gradients of the order of 5°C per 100 km and, if we accept 400 km as a likely dimension for the system we should expect temperature irregularities of the order of 10°C on the same scale as the wind pattern. On the other hand, the radiosonde record does not show such variations of temperature and although temperature irregularities of an amplitude up to 3 or 4°C are sometimes observed with a vertical scale of a kilometer or two, temperature irregularities are usually smaller. We therefore conclude that motion in the disturbances which we are studying is not primarily in geostrophic balance.

3. Some dynamical considerations

The theory of small perturbations is a useful guide to the dynamical processes at work in disturbances of various scales and will be applied here. A complete discussion of the types and sizes of disturbances which should be expected in a specified atmosphere requires detailed consideration of the boundary conditions to be applied to the problems. This will not be attempted because any boundary conditions posed for flow in the stratosphere are likely to be highly artificial. Instead the dimensions of the disturbance will be assumed known from the considerations of the previous section and perturbation theory will be used to demonstrate the possibility of such motions and the dynamical balance in them. The question of the compatibility of the motion with boundary conditions will not be considered. Whether or not the observed motions in the atmosphere are of a strictly wave-like character, the balance between the various terms in the equations of motion is likely to be similar to that occurring in the treatment of small sinusoidal disturbances.

Queney (1947) has given a comprehensive treatment of small oscillations in a stable tmosphere and has discussed the significance of damping of the wave motion by turbulent tresses, and by conductive and radiative transfer of heat, concluding that their effects are egligible. Here the latter two effects are ignored but terms representing the turbulent tress are retained in the equations because of their importance as the vertical scale of the isturbance is diminished.

Consider perturbations upon a zonal current in which the velocity u is a linear function f z. With the usual notation, and using pressure as a vertical coordinate, the equations f motions are :

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \varpi\frac{\partial u}{\partial p} - lv = -g\frac{\partial h}{\partial x} + K\frac{\partial^2 u}{\partial z^2} \quad . \quad . \quad (1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \varpi\frac{\partial v}{\partial p} + lu = -g\frac{\partial h}{\partial y} + K\frac{\partial^2 v}{\partial z^2} \quad . \quad . \quad (2)$$

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial \varpi}{\partial p} = 0 \quad . \quad . \quad . \quad . \quad (3)$$

The notation adopted here is as follows:

x, y = horizontal coordinates
p = pressure
u, v = horizontal components of velocity
$h(x, y, p)$ = height of an isobaric surface
g = gravity
K = eddy diffusivity
$\varpi \equiv dp/dt$
ρ = density
θ = potential temperature

In view of the small vertical accelerations of the air compared with gravity, the hydrostatic equation can be regarded as applicable. This has been assumed in the use of isobaric coordinates and will be justified later. From the hydrostatic equation we also deduce

$$\frac{\partial h}{\partial p} = -\frac{1}{g\rho} = -\frac{R\theta p^{-\frac{1}{\gamma}}}{g p_0^{\frac{\gamma-1}{\gamma}}}$$

The thermodynamic equation for adiabatic conditions then becomes

$$\frac{D}{Dt}\left(\frac{\partial h}{\partial p}\right) = -\frac{Rp^{-\frac{1}{\gamma}}}{gp_0^{\frac{\gamma-1}{\gamma}}}\frac{D\theta}{Dt} = -B\varpi \quad . \quad . \quad . \quad (4)$$

where

$$B = -\frac{R}{gp}\Gamma \text{ and } \Gamma = \frac{\partial T}{\partial p} - \left(\frac{\partial T}{\partial p}\right)_{\text{dry adiabatic}}$$

Here D/Dt ($\equiv u\,\partial/\partial x + v\,\partial/\partial y$) indicates differentiation following the horizontal movement of the air. (B is positive under conditions of static stability). Eq. (4) expresses the effect of vertical motion on temperature.

In order to consider perturbations upon the basic current, the following notation is used with the perturbation quantities indicated by ' primes ' :

$$u = U + u', \qquad v = v', \qquad \varpi = \varpi'$$

$$h = h(y, p) + h'(x, y, p)$$

Where U may be a function of p but not of x and y. Ignoring second-order quantities the perturbation equations become :

$$\left.\begin{aligned}
\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + \varpi'\frac{\partial U}{\partial p} - lv' &= -g\frac{\partial h'}{\partial x} + g^2\rho^2 K\frac{\partial^2 u'}{\partial p^2} \\
\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} + lu' &= -g\frac{\partial h'}{\partial y} + g^2\rho^2 K\frac{\partial^2 v'}{\partial p^2} \\
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial \varpi'}{\partial p} &= 0 \\
\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\frac{\partial h'}{\partial p} &= -B\varpi'
\end{aligned}\right\} \quad . \quad . \quad (5)$$

In the further discussion of these equations both U and $\partial U/\partial\rho$ will be treated as constants. This is equivalent to assuming that the wind shear is small enough for U to vary only by a small fraction of its value over one vertical wavelength of the disturbance.

In order to study the relative magnitude of the terms and the nature of the disturbance a sinusoidal variation is assumed in the x, y, p and t directions. Thus $u' = u^* \exp(i(\lambda(x - ct) + \mu y + \nu p))$, etc., where u^*, v^*, ϖ^* and h^* are independent of x, y, p and t. Eqs. (5) then reduce to :

$$i\lambda(U - c)u^* + \frac{\partial u}{\partial p}\varpi^* - lv^* = -ig\lambda h^* - g^2\rho^2 K\nu^2 u^* \quad . \quad . \quad (6)$$

$$i\lambda(U - c)v^* + lu^* = -ig\mu h^* - g^2\rho^2 K\nu^2 v^* \quad . \quad . \quad (7)$$

$$\lambda u^* + \mu v^* + \nu\varpi^* = 0 \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\nu\lambda(U - c)h^* = B\varpi^* \quad . \quad . \quad . \quad . \quad (9)$$

If λ, μ and ν are specified, these equations can be used to determine $(U - c)$ by elimination of u^*, v^*, ϖ^* and h^*. Thus writing $A = i\lambda(U - c) + g^2\rho^2 K\nu^2$

$$\begin{vmatrix}
A & -l & \frac{\partial U}{\partial p} & ig\lambda \\
l & A & 0 & ig\mu \\
\lambda & \mu & \nu & 0 \\
0 & 0 & B & i\nu(A - g^2\rho^2 K\nu^2)
\end{vmatrix} = 0$$

Expanding

$$\nu(A - g^2\rho^2 K\nu^2)\left\{(A^2 + l^2)\nu + \frac{\partial U}{\partial\rho}(l\mu - A\lambda)\right\} + gAB(\lambda^2 + \mu^2) = 0 \quad . \quad (10)$$

Before proceeding further it is convenient to specify values which are typical of the air stream and of the dimensions of the disturbances which we have observed.

Appropriate values are as follows :

Horizontal wavelength = 400 km	$\lambda = \mu = 1{\cdot}6 \times 10^{-7}$ cm^{-1}
Vertical wavelength = 1·5 km	$\nu = 1{\cdot}3 \times 10^{-4}$ g^{-1} cm^{-1} sec^{-2}
Approximately isothermal lapse rate	$gB = 4{\cdot}5 \times 10^{-3}$ cm^{-4} sec^{-2} g^{-2}
Shear of 2 m sec^{-1} km^{-1}	$\frac{\partial U}{\partial p} = 7 \times 10^{-3}$ g^{-1} cm^{-2} sec^{-1}
Eddy diffusivity – a value less than in inversion conditions in the lower layers over the sea seems appropriate.	$K = 10^3$ cm^{-2} sec^{-1}
Coriolis parameter	$l = 10^{-4}$ sec^{-1}

Returning to Eq. (10), if eddy viscosity is negligible ($K = 0$) and wind shear is absent, $\partial U/\partial p = 0$, we have three solutions for A, namely :

$$A = 0 \text{ and } A = \pm\left(-l^2 - \frac{gB(\lambda^2+\mu^2)}{\nu^2}\right)^{\frac{1}{2}}. \quad . \quad . \quad (11)$$

Substitution in Eqs. (6)-(9) of $A = 0$ shows that $\varpi^* = 0$ and

$$-lv^* = -ig\lambda h^*$$

$$lu^* = -ig\mu h^*$$

$A = 0$ implies that the disturbance pattern moves with the air when $K = 0$. The motion is geostrophic and this solution is not likely to be relevant to the present problem.

Substitution of values in the second expression (11) shows that the second term in the square root is a little larger than the first and $A = \pm 1{\cdot}5 \times 10^{-4}\, i\ \text{sec}^{-1}$ corresponding to $U - c = 10\ \text{m sec}^{-1}$. The solution for $U - c$ are necessarily real and the oscillations stable.

If we now compare the term arising from the eddy viscosity, namely :

$$g^2\rho^2 K\nu^2\ (\sim 2{\cdot}6 \times 10^{-5}) \text{ with } A\ (\sim 1{\cdot}5 \times 10^{-4}\, i)$$

we find that it is indeed substantially smaller and the character of the motion will not be greatly influenced by its presence.

Introduction of the shear term, $\partial U/\partial p \neq 0$, leads to solution of Eq. (10) of the form

$$A = \tfrac{1}{2}\left[\frac{\lambda}{\nu}\frac{\partial U}{\partial p} \pm \left\{\frac{\lambda^2}{\mu^2}\left(\frac{\partial U}{\partial p}\right)^2 - 4\left(l^2 + \frac{gB(\lambda^2+\mu^2)}{\nu^2} + \frac{\partial U}{\partial p}\frac{l\mu}{\nu}\right)\right\}^{\frac{1}{2}}\right]. \quad . \quad (12)$$

The introduction of shear introduces a real part into A and consequently introduces unstable waves into the solution. However, with the values specified above

$$\frac{\lambda}{\nu}\frac{\partial U}{\partial p} = \frac{\mu}{\nu}\frac{\partial U}{\partial p} \simeq \tfrac{1}{12}\, l$$

Consequently the solutions for A are little affected by introduction of the shear term.

It is thus possible to conclude that wave motion on the scale specified would behave similarly to stable waves with velocity relative to the stream $U - c \sim 10\ \text{m sec}^{-1}$, The waves might be slightly damped or amplified according as the shear or eddy viscosity dominates. (Rather different conclusions would be reached if $\lambda \gg \mu$, i.e., if the wavelength across the stream were much greater than along it. However, since the wavelength of the disturbances observed at Crawley is deduced to be at least 400 km, a solution with much greater wavelength in the perpendicular direction is hardly relevant).

Since $U - c$ is of the same order as the wind U at the level concerned and since U has not been previously specified, stationary waves with $c = 0$ are clearly possible and might be stimulated by orographic features. However, the topographic relief near Crawley is hardly sufficiently marked to make such stratospheric effects very likely.

If substitution is made from Eqs. (8) and (9) into Eq. (6), the following equation is derived which is convenient for examining the relative importance of the dynamical effects in producing the momentum changes.

$$i\lambda(U-c)u^* - \frac{\partial U}{\partial p}\frac{1}{\nu}(\lambda u^* + \mu v^*) - lv^* + \frac{iBg}{\nu^2(U-c)}(\lambda u^* + \mu v^*) + g^2\rho^2 K\nu^2 u^* = 0 \quad . \quad (13)$$

inertia (A)	vertical advection (shear) (B)	Coriolis (C)	thermodynamical (D)	frictional (E)

The effect represented by each term is indicated.

In deriving (12) the hydrostatic condition was assumed. The third equation of motion which should replace the hydrostatic equation is

$$\frac{\partial p}{\partial h} = -g\rho - \rho\frac{dw}{dt}$$

Thus in estimating $\partial h/\partial p$, by omitting the second term on the right, an error is made of the order of

$$\frac{1}{g^2\rho}\frac{dw}{dt} \sim \frac{1}{g^3\rho^2}\frac{d\varpi}{dt}$$

$$\sim \frac{1}{g^3\rho}\lambda(U-c)\varpi^*$$

Thus the error in Eq. (13) when substitution is made from Eq. (9) is of the order of

$$\frac{\lambda^2(U-c)\varpi^*}{g^2\rho^2\nu} \sim \frac{\lambda^3(U-c)u^*}{g^2\rho^2\nu^2}$$

With the magnitudes suggested above, the relative size of the terms in Eq. (12) can be estimated as follows :

$1{\cdot}6\times10^{-4}u^*$	$10^{-5}u^*$	$10^{-4}u^*$	$0{\cdot}7\times10^{-4}u^*$	$2{\cdot}6\times10^{-5}u^*$	cm sec^{-2}
(A)	(B)	(C)	(D)	(E)	

and the hydrostatic error term is of the order $1{\cdot}6\times10^{-10}$ cm sec^{-2}.

It is clear, therefore, that the possibility of oscillatory motion arises from the balance between the Coriolis, inertia and thermodynamic terms. None of these is negligible. Inertia waves in which the primary balance is between the Coriolis and inertia terms have been extensively discussed in the theoretical literature (see, for example, van Mieghem 1945). In discussing these waves the thermodynamic term is commonly ignored and the shear term included but the motion essentially similar in character to that discussed here.

The possible dimensions of such inertial disturbances are rather closely limited. Thus if the vertical dimension is decreased (ν increased) the magnitude of the frictional term (E) increases inversely with the square of the vertical wavelength. Thus any substantial decrease in the vertical scale soon introduces important modifying frictional effects.

If the vertical wavelength is increased substantially the inertia and thermodynamic terms are increased substantially relative to the Coriolis term, the velocity of the waves is also increased and their character changed. A decrease in horizontal wave length has a similar effect. An upper limit to the horizontal wavelength is probably imposed by the extent of a quasi-uniform basic stream.

4. Conclusions

The observational records of irregularities of wind in the stratosphere persisting over periods of hours, provide evidence for the existence of dynamical disturbances of the motion in the stratosphere in which the primary balance is between the inertia, Coriolis effects and the static stability. These disturbances necessarily have a vertical scale of the order of a kilometre or two and a horizontal scale of the order of hundreds of kilometres.

Such inertia oscillations had not been previously observed except in the tracks of constant-level balloons (Neiburger and Angell 1956). This is probably because of their relatively small vertical scale which is less than the vertical range over which most synoptic observations are smoothed in the stratosphere.

Fast-moving non-advective waves in the upper troposphere have been reported by Hess and Wagner (1948) over the Rockies. These are, however, accompanied by significant surface-pressure variations and extend through a deep layer of the atmosphere. It is possible

that these are, in fact, somewhat similar 'inertia waves' with a balance of factors rather similar to that discussed here. However, at Crawley, the vertical irregularity of the wind is less well-developed in the troposphere than the stratosphere and this may perhaps be attributed to the larger value of the eddy diffusivity K or to the less uniform and less extensive nature of the basic current.

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My attention was first directed to the quasi-periodic features of the Crawley radar-sonde records by Mr. A. L. Maidens. I am indebted to him for this and to the Meteorological Office staff at Crawley and in the Instrument Development Branch who made the observations.

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Some experiments in the objective analysis of contour charts

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SUMMARY

The quality of a series of objective analyses of synoptic charts has been examined and it has been found that statistically the discrepancies from manual analyses are of the same order as the variations which arise amongst conventional analyses by different individuals. A systematic fault in the computed analyses of surface maps (viz. an underestimation of the depth of intense depressions over the ocean) was found to be due to the use of the geostrophic approximation and to other limitations of the interpolation system. Modifications designed to overcome this shortcoming have raised the standard of the analysis of depressions to that of the analyses as a whole.

1. INTRODUCTION

During 1959 experiments in numerical forecasting were carried out using the electronic computer METEOR at the Meteorological Office, Dunstable. Part of these experiments comprised the computation of objective analyses of the fields of contour height required as initial data for the numerical forecasts. The objective analyses were required to provide values of 500 mb contour height and 1,000-500 mb thickness at each of 480 grid points, the values being obtained by interpolation between the observations provided by the radiosonde network. In addition, a similar objective analysis of surface observations of wind and pressure from ships on the Atlantic was computed and this surface analysis was exploited to improve the detail and accuracy of the upper fields.

The objective analyses were fairly satisfactory with the merit for numerical forecasting that they were smoother than the corresponding subjective analyses and accordingly were less liable to give rise to computational instability. However, an examination of individual cases revealed that the depth of intense depressions on the Atlantic was systematically underestimated in the objective analyses. This paper is concerned with the diagnosis of the reasons for this shortcoming of the objective analysis procedure and with the attempts to rectify it.

2. THE BASIC METHOD

Observations of 500 mb height and wind from a suitable network of upper-air stations were read into the computer's memory, and were analysed using the method described by Bushby and Huckle (1957). In fact the thickness field was analysed concurrently in the same way, but for our purpose here we need only consider one field. Briefly, each grid point was dealt with individually and a quadric surface of the form

$$h_s = ax^2 + 2\,hxy + by^2 + 2\,gx + 2\,fy + c \quad . \quad . \quad (1)$$

was fitted by least squares to the observations within the vicinity of each grid point. Here h_s is identified with the 500 mb height and x, y are position coordinates on the grid. The geostrophic approximation provided an estimate of the slope of the surface in the vicinity of each observation and it was thus possible to arrange for the fitting of wind as well as height observations in determining the best-fitting quadric. Furthermore, in order to introduce an element of time continuity into the analysis and to ensure a solution in regions of sparse data, a forecast field (normally a 24-hour numerical forecast) was also introduced into the process. On this basis, the expression which was minimised to determine the quadric took the form

$$E = \Sigma p\,(h_s - h_0)^2 + T^2\,\Sigma p\,(\mathbf{V}_s - \mathbf{V}_0)^2 + \Sigma q\,(h_s - h_f)^2 \quad . \quad . \quad (2)$$

where

p, q = distance weighting functions of the form $1/(1 + k r^4)$, r being the distance between observation and grid point, and k a constant.

$h_s, \mathbf{V}_s$ = the height and geostrophic wind at (x, y) given by the quadric Eq. (1).

$h_0, \mathbf{V}_0$ = observed values of height and wind.

h_f = forecast height at a grid point.

T^2 = parameter chosen to give suitable weight to the wind observations relative to the heights.

Values of p, q, T^2 as determined previously from experiments described by Bushby and Huckle (1957) were used, and the summations were effected over an appropriate number of observations (up to six) and forecast values (nine). The quadric of best fit was found by forming and solving the six linear equations

$$\frac{\partial E}{\partial a} = \frac{\partial E}{\partial h} = \frac{\partial E}{\partial b} = \frac{\partial E}{\partial g} = \frac{\partial E}{\partial f} = \frac{\partial E}{\partial c} = 0$$

obtained by partial differentiation of Eq. (2). The value of h_s given by the quadric at the grid point was taken to be the interpolated 500 mb height there.

Although various checks were applied to the data during its extraction a further attempt was made to detect spurious data as follows. The observed heights and winds were compared with values interpolated from the objective analysis. Discrepancies greater than certain amounts were taken to indicate erroneous observations which were then erased from the computer's memory; the analysis was then repeated. The process was successful in areas of plentiful observations but was not relied upon over the ocean areas for obvious reasons. Cressman (1959) describes a similar method which has been used at the Joint Numerical Weather Prediction Unit, Washington.

The analyses produced by the method as so far described, being based only on upper-air observations, were unduly smooth and lacking in detail over the Atlantic Ocean where the upper-air network is sparse. The human analyst overcomes this difficulty to a considerable extent by exploiting surface observations from ships, ensuring by a graphical technique that his charts for various levels have vertical consistency and do not contravene the hydrostatic approximation. This has the effect of improving both the accuracy and the representation of detail in the upper charts over sparse networks. The process was simulated to a large extent in the computed analyses as follows.

An analysis of the surface observations of wind and pressure (converted to 1,000 mb height) from ships on the Atlantic was computed using the quadric fitting process for each grid point as already described. In place of the forecast field a good first guess was provided by the difference between the 500 mb and total thickness analyses already computed from the upper-air data alone. The surface wind may depart substantially from the geostrophic value and, indeed, in some circumstances, may even deviate towards high pressure. However, as an approximate mean allowance for the effect of the friction layer the surface-wind observations were veered by 10°, and increased by 25 per cent. A procedure for the elimination of spurious observations, similar to that described for the upper-air data, was included.

Having determined a revised surface analysis in this way it was next necessary to modify the previously found 500 mb and thickness analyses so as to achieve vertical consistency with the new and improved surface analysis. In the nature of the problem there is no unique procedure for deciding what consequential adjustments should be made to the upper fields, and a rather arbitrary procedure was adopted for the time being. Of the adjustments to the surface pattern required to accommodate the surface observations, half were assigned to the 500 mb field and half to the total thickness. It was found subsequently, however, that the closest approach to the charts analysed by hand was achieved by imparting most of the additional detail to the 500 mb chart, leaving the thickness chart smooth.

3. The accuracy of the analyses

Over the land areas the computed analyses differed very little from those drawn by hand, especially over Europe where the station network is the most dense. This is to be be expected as the network of observations over most of the land areas within the grid in use is adequate to define synoptic scale systems with very little latitude. A much greater degree of uncertainty is inevitable over the oceans, however, and we can no more expect precise agreement between the computed and conventional analyses than we could between those produced independently by different individuals.

Some statistics expressing the difference between the computed and corresponding conventional analyses, evaluated over 92 situations, are given in Table 1.

TABLE 1. Analysis comparisons

	h_e (metres)	V_e (knots)
1,000 mb	20	13
1,000-500 mb thickness	23	12
500 mb	25	13

Root mean square height (h_e) and geostrophic wind (V_e) discrepancies between computed and conventional analyses (92 cases), evaluated over 150 grid points covering Western Europe and the Eastern Atlantic.

The aim in an objective analysis system must presumably be to achieve results which differ from the manual product by no more than the human variation. Precise figures expressing this aim in terms of the grid in use are not available but we may note that Berggren (1957) found from 29 analyses of a 500 mb chart by different individuals a standard deviation of 500 mb height of 15 m indicating a r.m.s. difference between any two analyses of 21 m. The latter figure was determined for a somewhat larger grid but it represents differences of the same order of magnitude as those of Table 1. We conclude that our computed analyses must in most respects be reasonably good. That this was not so in all respects was made abundantly clear by an examination of the individual analyses over the oceans.

The examination revealed that the depth of depressions on the Atlantic was systematically underestimated by the objective method, particularly for intense systems. Indeed the r.m.s. difference between the computed and subjective values of contour height at the centre of depressions on the Atlantic was 66 m (8·1 mb), evaluated over 71 depressions. Furthermore, the error was invariably in the same sense, viz., an underestimation of the depth in the computed analysis. Undoubtedly the inherent smoothing of the objective technique, which is necessary if spurious short-wave irregularities in the vicinity of isolated observations are to be avoided, was a contributory factor. Nevertheless the magnitude of these errors suggested some more fundamental shortcoming of the objective method.

4. Cause of the analysis shortcomings

Consideration of the nature of the main analysis fault suggested that it might be due to two causes :

(1) The quadric surfaces which can be represented by Eq. (1) are limited to elliptic or hyperbolic paraboloids having one axis vertical. In particular, hyperboloids and cones are excluded and all vertical sections are necessarily parabolae with vertical axes. A parabola has a characteristic rounded shape in the vicinity of the vertex and cannot assume a pointed shape there unless the latus rectum is small. It seemed possible that this might account at least partially for the truncation of the profiles of intense depressions*.

* I am indebted to Dr. E. J. Langham for suggesting this possibility.

(2) Estimates of the slope of the pressure surface were used in the fitting process, these being obtained from the observed winds by means of the geostrophic approximation. It is well known, however, that observed wind speeds may be considerably less than the geostrophic values in regions of large cyclonic curvature, and clearly this would have the effect of flattening the profiles of intense depressions in the computed analyses.

In order to examine these effects an experiment was set up using an idealized circular depression, the diameter of the contour of zero 1,000 mb height being four grid lengths and the depth at the centre being -400 m. The profile was specified analytically in such a form that it could be made to assume a parabolic or conical shape and any hyperbolic shape between these extremes. Six computed values of height and geostrophic wind exactly fitting the idealized depression were used as artificial observations and a least-squares fit of the surface given by Eq. (1) was computed as in the standard analysis system described. It was found that as the profile of the depression was made to depart more and more from a parabolic shape, the inability of the quadric given by Eq. (1) to provide a good fit in the middle of the depression became more and more pronounced. In the extreme case of a conical profile the error at the centre was 114 m; for a moderately hyperbolic profile the error at the centre was about 50 m.

The computations were repeated using the same six artificial height observations but using gradient winds computed for the hyperbolic depression profile as the wind observations. This simulates what occurs in the practical analysis scheme when observed winds in the region of a depression although sub-geostrophic are used to derive contour gradients by means of the geostrophic equation. In this case, and as one would expect, the error of fit at the centre was even greater, being about 100 m for a moderately hyperbolic profile.

A number of similar computations confirmed that both the restrictions inherent in the use of Eq. (1), and in the use of observed winds through the geostrophic equation, must be sources of appreciable error in the objective analysis of intense depressions.

It is simple to devise modifications to the system which would be the formally correct method of overcoming these difficulties. For example, if a term in h_s^2 is added to Eq. (1), the vertical cross-sections of the surface it represents could then be hyperbolic or even conical. However, h_s in Eq. (2) would have to be expressed as the solution of a quadratic in h_s, leading to seven highly non-linear algebraic equations in the coefficients of the quadric. As the method does not appear practicable, several other major difficulties which also arise need not be discussed. The obvious technique for avoiding use of the geostrophic approximation, namely to insert for $\mathbf{V}_s$ in Eq. (2) an expression for the gradient wind at position (x, y) on the quadric surface, also leads to formidable difficulties. It would be unrealistic to introduce such complications in a computation which must be repeated a large number of times to achieve a complete analysis and it was necessary to devise simpler alternatives.

5. Allowance for curvature of the flow

The gradient wind equation may be written

$$\frac{V_{\text{geo}}}{V_{\text{gra}}} = 1 + \frac{V_{\text{gra}}}{rf} \qquad . \quad . \quad . \quad . \quad (3)$$

where r is the radius of curvature of the flow, treated as positive for cyclonic curvature, and f is the Coriolis parameter. The expression on the right-hand side of Eq. (3) was evaluated for each wind observation, identifying the observed wind with the gradient wind in the expression. The gradient of contour height derived from the wind for use in the fitting process was then multiplied by the factor so obtained. This necessitated a knowledge of the curvature of the flow, and for this the curvature of the contours, computed from a first analysis, was used as an approximation, the gradient adjustment being

applied during a second analysis. The success of the scheme depends on the assumption that the main fault of the basic analysis system, viz., an underestimation of the depth of depressions, would have little effect on the field of curvature of the contours.

Safeguards were incorporated in the computation to prevent absurd adjustments being made fortuitously and it was found that typically, e.g., at about three grid lengths from the centre of a substantial depression, the procedure described above resulted in an increase of about 50 per cent in the contour gradients derived from the winds. The effect on the analyses appeared to be entirely beneficial and of the total improvement achieved in the analyses and reported quantitatively later in this paper, about 30 per cent was attributable to this approximate allowance for the curvature of the contours.

6. Allowance for the limitations of the quadric surface

If an observation of contour height (h_0) and wind (components u, v) is available at position (x, y) relative to a nearby grid point, then we can extrapolate using the geostrophic equation to obtain at the grid point the value

$$h = h_0 + \frac{f}{g\beta}(uy - vx) \quad . \quad . \quad . \quad . \quad (4)$$

where β is the map-scale factor.

If several such observations are available, a weighted mean of these values at the grid point can be obtained in the form

$$h_p = \frac{\Sigma\,[p\,\{h_0 + f(uy - vx)/g\beta\}]}{\Sigma p} \quad . \quad . \quad . \quad (5)$$

p being a distance weighting function. The value given by Eq. (5) when used for extrapolation to the centre of a depression from observations around it would generally yield a value lower than that provided by the quadric fitting process. It seemed likely, therefore, that a weighted mean of these two values might be better than either of them individually. If the quadric process yielded a value h_Q the weighted mean would have the form

$$H = \frac{h_Q + K h_p}{1 + K} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where K remains to be determined.

Clearly we can rely more heavily on the value obtained by direct extrapolation from the observations when these are plentiful and/or near the grid point. Accordingly, a reasonable choice for the form of K would be

$$K = \frac{\Sigma p}{(\Sigma p)_{\max} - \Sigma p} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where (Σp) max is the greatest value which Σp could assume, i.e., when the full number of observations (six) used in dealing with any one grid point were all available and close to it.

This process was tried out for the analysis of a number of situations with intense depressions on the Atlantic, the analyses of which had proved quite unsatisfactory with the quadric fitting process. The form of distance-weighting function (p) used is illustrated by the full curve in Fig. 1; it will be noted that the sharp cut-off in p occurs for observations distant from 2 to 4 grid lengths from the relevant grid point. One example is illustrated in Figs. 2 (a), (b), (c). The conventional analysis of the depression in which we are interested is given in Fig. 2 (a) where the contours are of 1,000 mb height, drawn at intervals of 30 m. The analysis obtained from the ordinary quadric fitting process, Fig. 2 (b),

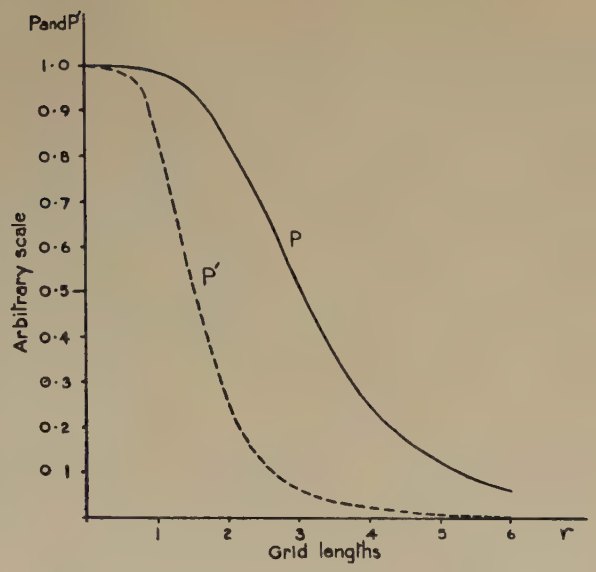

Figure 1. Forms of distance weighting function used in the analysis process (see text).

ndicates the magnitude of the analysis error in the central part of the low, the discrepancy ield being shaded at 30 m intervals. The r.m.s. discrepancy between the computed and manual analyses evaluated over the depression (approximately within the contour of zero ,000 mb height) was 31 m whilst the error in the computed analysis of the depth of the ow was 150 m; there are five contour lines absent from the central part of the computed nalysis. It can be seen from Fig. 2 (c) that the analysis computed using the quadric itting process modified as described resulted in a substantially deeper centre; indeed, our additional contour lines have been gained in the central area and the error in the nalysed depth has been reduced to about 20 m. Unfortunately, it will also be seen from he shaded discrepancy field on Fig. 2 (c) that the modified process has resulted in considerable distortion of the depression, with regions of negative discrepancy (analysis too eep) especially in the NW quadrant of the low. The r.m.s. discrepancy of the modified nalysis was in fact increased to 38 m in spite of the improvement in the analysed depth of the depression.

Similar results were obtained in a number of other cases, although the magnitude nd location of the distortions introduced varied with the number and distribution of the vailable observations.

It was not difficult to discover the reasons for these shortcomings. They arise because he isobaric surface in the region of an intense depression can only be represented by a plane over a very limited area. Thus extrapolation of contour height away from an observaion, using the contour gradient indicated by the observed wind, gives grossly erroneous values if in fact the gradient reverses or changes direction substantially, as occurs in depressions. In particular, with the present process an excess of observations on one side of the depression tends to cause the contour heights on the opposite side to be lowered unduly, and the analysed position of the centre is displaced away from the region of most plentiful observations. Furthermore, aside from the question of extrapolation across the centre of a depression, it is more risky to extrapolate away from an observation along the wind vector than perpendicular to it. The error field of the analysis in Fig. 2 (c) arises from factors of this type, and it is clear that for the present process to yield acceptable results a rather subtle balance between h_Q and h_p in Eq. (6) is required. After considerable experimenting, the following additional modifications to the process were introduced.

Figure 2. (a) Manual analysis of intense depression on the Atlantic ocean, 00 GMT, 16 January, 1959. 1,000 mb contours at 30 m intervals. Contours 0 m, − 150 m and − 300 m thickened. (b) Computed analysis of depression using ordinary quadric fitting process. Discrepancy field from human analysis indicated by shading. (c) Computed analysis using trial modifications to process (see text). (d) Computed analysis using further modifications (see text).

(a) A revised distance weighting function p' was adopted for the computation of h_p. This is indicated by the dashed curve in Fig. 1. This restricts further the range of effect of the observations in the plane-fitting part of the process.

(b) The full value p' was used for extrapolation perpendicular to the wind vector. For extrapolation in other directions from the observations, an additional restraint was added by multiplying p' by the factor $1/(1 + 2r^2 \cos^2 \gamma)$ where γ is the angle between the wind vector and the line joining observation to grid point. The effect of this modification is to make the isopleths of equal weighting

function circular for small ranges and increasingly oval as the range increases. Less attention is thus paid to the value obtained at the grid point if this arises from extrapolation in a direction well removed from the perpendicular to the wind vector.

(c) K in Eq. (6) was multiplied by an additional factor :

$$K' = \frac{150}{|h_Q - h_p|} \quad . \quad . \quad . \quad . \quad (8)$$

when $K' < 1$; otherwise no adjustment was made. This was designed to restrict the adjustment made to h_Q by Eq. (6) in cases where h_p was departing very substantially from h_Q as, for example, when the extrapolation was across the centre of a depression.

These additional devices were tried out individually and systematically and were all found to be beneficial. The total effect on the particular depression already discussed can be seen from Fig. 2 (d) where the r.m.s. discrepancy from the manual analysis has fallen to 21 m. The analysis error at the centre, however, (60 m) remains rather high in this case but the present process does not appear capable of completely curing this fault.

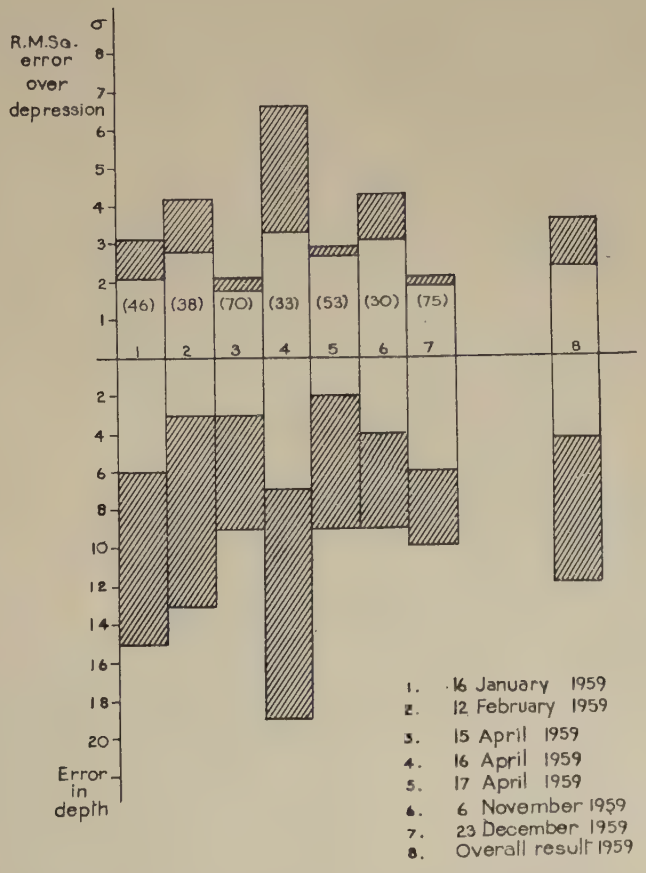

Figure 3. Root mean square analysis discrepancies over seven intense depressions on the Atlantic ocean and analysis discrepancies at the centres. Errors for the ordinary quadric fitting process and the modified process are shown, the shaded areas representing the improvement achieved. Number of grid points shown within histograms.

7. Some summarized results and conclusions

The analysis process described above was applied to six further situations all of which included very deep depressions on the Atlantic ocean. A useful degree of improvement as compared with the ordinary quadric fitting process, was achieved in every case. The results are summarized in Fig. 3 in the form of r.m.s. discrepancies from the corresponding manual analyses. These discrepancies were evaluated only over the depressions, roughly within the contour of zero 1,000 mb height. The analysis errors at the centres are also illustrated. For comparison, the corresponding results for the ordinary quadric fitting process are also given, the shaded area indicating the improvements achieved by means of the additional devices introduced into the computation. Overall, the modified process has resulted in a reduction in the root mean square discrepancy from 35 m to 24 m and a reduction in the analysis error at the centre from 125 m to 45 m.

Thus the r.m.s. analysis discrepancy, evaluated over a number of intense depressions on the ocean, has been brought to a level about the same as that achieved over whole charts (see Table 1). So far as is known, objective analyses of surface maps have not yet been attempted elsewhere but we may note that the discrepancy figure for our intense depressions, 24 m, compares favourably with values of 26 m and 24 m quoted by Bergthorssen and Döös (1955) as the r.m.s. errors of 500 mb analyses computed in Sweden. These figures by Bergthorssen and Döös benefited from the use of a 12-hr forecast as the first estimate of the analyses; in our case 24-hour numerical forecasts were used and it is known that this lowers the standard of the final analyses. We may also note the figure of 26 m given by Bergthorssen and Döös (1955) as the r.m.s. discrepancy between a series of 500 mb charts analysed independently by two different individuals.

In considering the import of these figures we must bear in mind that a r.m.s. discrepancy is a very stringent test of an analysis. Cressman (1959) has quoted r.m.s. discrepancies of fit at the observations for 500 mb analyses computed at the Joint Numerical Weather Prediction Unit, Washington, rather than discrepancies at the grid points from manual analyses. Although it is of course of interest to know how close the observations are fitted by the process used, such figures do not reveal how great may be the discrepancy from the conventional analysis over the regions of the charts where there are little or no data.

These considerations and the comparison figures given lead us to conclude that the analysis process described yields results which approach the optimum attainable.

Acknowledgments

I am indebted to Mr. P. Graystone for assistance of various kinds and to Miss M. Sherwin, who took a substantial part in the machine computations.

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551.558 : 551.511.32

The generation of wave energy at a fluid interface by the passage of a vertically moving slender body

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Summary

In this paper an estimate is made of the energy absorbed by wave motion at the interface of two superposed fluids when a body passes from one fluid to the other. The fluids are supposed perfect, incompressible, and bounded only by the interface, and axi-symmetric and symmetrical two-dimensional cases are considered. The body is supposed to move vertically and approach the interface from a great distance, and after crossing the interface to recede to a great distance, the speed being kept steady throughout. It is assumed that there were no waves originally present at the interface and that no splashing occurs during the crossing. The linear approximation to irrotational waves is used at the interface, and the slender body approximation is used at the boundary of the body. Transform methods are used to solve the Laplace equation for the velocity potential. Expressions are found for the waves set up as the body crosses the interface, and the energy used to generate the waves is found from a calculation of the work done against the wave resistance. The wave energy may be regarded as the sum of the wave energies for two cases : one in which the density difference between the two fluids is small, and another, in which the density of the upper fluid is zero. For the special cases of a spheroid and a spindle-shaped body, curves are shown of the wave energy against a dimensionless form of the inverse speed. Criteria are given for when maximum wave energy is developed for a thermal, and an estimate of this energy is made. This is done in Section 6.

1. Introduction

The problem arises in the study of the ascent of thermals through the atmosphere. During the ascent, the thermal acquires kinetic energy derived from its potential energy by the mechanism of the buoyancy, and subsequently much of this energy is dissipated into turbulent eddies within the thermal itself. However, if the surrounding air layers are disturbed into wave motions by the passage of the thermal, energy is radiated away from it by them and this provides another means by which the thermal loses energy. This action involves a dispersion rather than a dissipation of energy. Although eventually all the kinetic energy is dissipated by viscosity, the wave motions spread the energy over a much larger region than that of the thermal and its immediate surroundings. Here the eddy motion is predominant.

An estimate of the amount of energy dispersed into waves enables an estimate to be made of the effect on the thermal. For this purpose the thermal is regarded as a local region of turbulence such that if we move with the thermal during its ascent, this region is bounded by some closed surface, outside of which the motion of the air is well behaved. Both a steady speed of ascent and a fixed shape of the surface which confines the turbulence are assumed. In this paper we are interested in the case when the thermal rises through an inversion. An inversion is taken here to mean a thin layer of air where the temperature changes rapidly, so that the temperature may be regarded as discontinuous at the height of the layer. For these purposes the atmosphere may be regarded as an incompressible fluid whose density is equal to the reciprocal of the potential temperature, and an inversion is represented by a discontinuity in density, i.e., by a fluid interface. This analogy is well known and is discussed, for example, by Scorer (1952). The problem of how much wave energy is developed as a thermal ascends through an inversion corresponds to the case when the density difference between the two superposed fluids is small. The two-dimensional case of the problem occurs when a bent-over, buoyant chimney plume meets an inversion. The case which occurs when the upper fluid is absent or has negligible density has no application to the problem of the thermal rising through an inversion,

but it is included since this case is as readily handled. The theory assumes that the thermal crosses the interface without appreciable loss of speed. Subsequently the speed may change considerably, but this will not affect the result since most of the wave energy is developed when the thermal crosses the interface. Once an estimate of the wave energy is obtained, a comparison may be made with the kinetic energy of the thermal or with the work done in crossing the interface against the hydrostatic pressure. This comparison then gives a measure of the extent to which the generation of waves affects the motion of the thermal.

Fig. 1 (Plate I) is a photograph taken by Mr. J. M. Richards during one of his experiments at Imperial College on the behaviour of thermals. It shows a wave crest at an interface which has been induced by the passage of a (two-dimensional) thermal.

2. The axi-symmetric case

The boundary conditions

For the axisymmetric case, cylindrical coordinates are used, with the following notation :

r — horizontal coordinate

z — height. The z axis is coincident with that of the body

(u, w) — velocity of a fluid particle

ϕ — velocity potential, where

$$u = -\frac{\partial \phi}{\partial r} \text{ and } w = -\frac{\partial \phi}{\partial z}.$$

The velocity potential ϕ describes the motion in both upper and lower fluids. Thus $\phi = \phi$ (lower fluid) if $z < 0$, and $\phi = \phi$ (upper fluid) if $z > 0$.

W — steady vertical velocity of the body, positive upwards

a — half-width of the body

b — half-length of the body

ρ_2 — density of the lower fluid

$\rho_1 = \rho_2 - \Delta\rho$ = density of the upper fluid

$\rho_2 - \delta\rho$ = density of the thermal

$\sigma = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$

$\nu = gb\sigma/W^2$

The undisturbed interface is taken to be the surface $z = 0$, and the centre of the body is supposed to cross the surface at $t = 0$. The body is replaced by line sources and sinks distributed along the z-axis, and the strength of these sources and sinks may be written as

$$-2\pi W f(z - Wt) f'(z - Wt). \quad . \quad . \quad . \quad . \quad (1)$$

The accent denotes partial differentiation with respect to z. If we travel along the z-axis with a velocity W the sources have a relatively steady strength in a stream of velocity W and the separating stream lines represent the surface of a body in the usual way. At points near the z-axis

$$u \simeq -\frac{W}{r} f(z - Wt) f'(z - Wt), \quad . \quad . \quad . \quad (2)$$

or in terms of the velocity potential

$$\lim_{r \to 0} \left(r \frac{\partial \phi}{\partial r} \right) = W f(z - Wt) f'(z - Wt) \quad . \quad . \quad (3)$$

Figure 1 (Plate I). A wave crest induced by the passage of a (two-dimensional) thermal through an interface (inversion). The density above the interface is that of fresh water (1·000), and below equal to 1·001. The thermal was released from the double trough seen at the bottom and at the moment of the picture had a width about twice the distance between the black squares. It was composed of ammonia solution and a dye, and the density variations in it can be seen in the top corners of

If the axisymmetric body whose shape is given by $r_a = f(z)$ is a slender one so that $r_a = f(z)$ is small, then Eq. (2) gives

$$-\left(\frac{u}{W}\right)_{r=r_a} \doteq f'(z - Wt)$$

Hence, if we travel along the z-axis with a velocity W

$$\left(\frac{u}{w - W}\right)_{r=r_a} = f'(z) \quad . \quad . \quad . \quad . \quad (4)$$

approximately if $|w| \ll |W|$. But Eq. (4) is the condition that $r_a = f(z)$ is a fixed boundary in a stream of velocity $-W$. Hence the source and sink distribution whose strength is given by Eq. (1) is equivalent to a body which moves along the z-axis with a velocity W and whose shape is given approximately by $r_a = f(z)$ if it is slender. The accuracy of this approximation is discussed, for example, by Temple (1958).

At the interface the pressure is continuous, and hence, with the usual linear approximations,

$$\rho\left(\frac{\partial^2\phi}{\partial t^2} + g\frac{\partial\phi}{\partial z}\right) \text{ is continuous at } z = 0 \quad . \quad . \quad . \quad (5)$$

as is shown by Stoker (1957, pp. 21-22). To the same degree of approximation, the vertical velocity of the fluid particles at $z = 0$ is continuous, and hence

$$\frac{\partial\phi}{\partial z} \text{ is continuous at } z = 0 \quad . \quad . \quad . \quad . \quad (6)$$

Since Eqs. (5) and (6) involve linear approximations, no attempt is made to refine the slender-body approximation (1).

3. Calculation of the velocity potential

The equation satisfied by $\phi(r, z, t)$ is the Laplace equation in cylindrical polar coordinates:

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad . \quad . \quad . \quad . \quad (7)$$

This equation is transformed by means of the Hankel transform

$$\bar{\phi}(k, z, t) = \int_0^\infty dr \, . \, r J_0(kr)\, \phi(r, z, t) \quad . \quad . \quad . \quad (8)$$

If we multiply Eq. (7) by $r J_0(kr)$ and integrate with respect to r from ϵ to ∞ and let ϵ tend to zero we obtain

$$-k^2\bar{\phi} + \frac{\partial^2\bar{\phi}}{\partial z^2} = \lim_{r\to 0}\left(r\frac{\partial\phi}{\partial r}\right) \quad .$$

Here we assume that $\lim_{r\to\infty}(r\,\partial\phi/\partial r) = 0$ and that $\lim_{r\to\infty}(r^{\frac{1}{2}}\phi) = 0$. Therefore

$$-k^2\bar{\phi} + \frac{\partial^2\bar{\phi}}{\partial z^2} = Wf(z - Wt)f'(z - Wt), \quad . \quad . \quad (9)$$

by Eq. (3).

For convenience we replace the r.h.s. of Eq. (9) by

$$\frac{W}{4\pi i} m F(m) e^{-im(z - Wt)}$$

solve for $\bar{\phi}$, and then integrate with respect to m from $-\infty$ to ∞. Here

$$F(m) = \int_{-\infty}^{\infty} dz \{f(z)\}^2 e^{imz} = \int_{-b}^{b} dz \{f(z)\}^2 e^{imz}, \quad . \quad . \quad (10)$$

so that

$$\int_{-\infty}^{\infty} dm \frac{W}{4\pi i} m F(m) e^{-im(z-Wt)} = W f(z - Wt) f'(z - Wt).$$

For example, for a spheroid

$$f(z) = a\sqrt{1 - \left(\frac{z}{b}\right)^2} \text{ and } F(m) = \frac{4a^2}{b^2}\left(\frac{\sin mb - mb \cos mb}{m^3}\right).$$

The solution of Eq. (9) which satisfies the condition Eq. (6) may then be written

$$\bar{\phi} = \frac{iW}{4\pi} \cdot \frac{mF(m)}{k^2 + m^2} e^{-ik(z-Wt)} + (\text{sgn } z)\, G(k, m, t)\, e^{-k|z|}, \quad . \quad . \quad (11)$$

where sgn $z = +1$ if $z > 0$, and sgn $z = -1$, if $z < 0$. $G(m, k, t)$ is some function of k, m, and t, and is determined from the boundary condition (5) which gives

$$\rho_1 \left(\frac{\partial^2 \bar{\phi}}{\partial t^2} + g \frac{\partial \bar{\phi}}{\partial z}\right)_{z=+0} = \rho_2 \left(\frac{\partial^2 \bar{\phi}}{\partial t^2} + g \frac{\partial \bar{\phi}}{\partial z}\right)_{z=-0}.$$

The substitution in this equation for $\bar{\phi}$ as given by Eq. (11) gives

$$\frac{\partial^2 G}{\partial t^2} + g\sigma k\, G = \frac{W}{4\pi} \cdot \frac{m^2 (g - iW^2 m)}{k^2 + m^2} F(m) e^{imWt}.$$

Hence

$$G = \frac{W\sigma}{4\pi} \cdot \frac{m^2 (g - iW^2 m) e^{imWt}}{(k^2 + m^2)(gk - W^2 m^2)} + d_1(k, m) \cos \sqrt{g\sigma k}\, t + d_2(k, m) \sin \sqrt{g\sigma k}\, t,$$

where d_1 and d_2 are functions of k and m and may be determined from the initial condition that no waves are originally present. If $G = G_0$, say, when $g = 0$, then

$$G_0 = \frac{iW\sigma}{4\pi} \cdot \frac{mF(m) e^{imWt}}{k^2 + m^2}.$$

G may be written $G = G_0 + (G - G_0)$, where $(G - G_0)$ represents a term due to gravity. If we now integrate w.r.t. m from $-\infty$ to ∞ and then use the inverse of transform (8), we obtain

$$\phi = \phi_1 + \phi_2 + \phi_3 + \phi_4,$$

where

$$\phi_1 = \frac{iW}{4\pi} \int_0^{\infty} dk \int_{-\infty}^{\infty} dm \frac{k J_0(rk)\, m F(m) e^{-im(z-Wt)}}{k^2 + m^2},$$

$$\phi_2 = (\text{sgn } z) \cdot \frac{iW\sigma}{4\pi} \int_0^{\infty} dk \int_{-\infty}^{\infty} dm \frac{k J_0(rk)\, mF(m) e^{imWt - |z|k}}{k^2 + m^2},$$

$$\phi_3 = (\text{sgn } z) \cdot \frac{g\sigma}{4\pi W} \int_0^{\infty} dk \int_{-\infty}^{\infty} dm \frac{k J_0(rk)\, (i\sigma k - m) mF(m) e^{imWt - |z|k}}{\left(k^2 + m^2\right)\left(m^2 - \frac{g\sigma k}{W^2}\right)},$$

and

$$\phi_4 = (\text{sgn } z) \cdot \frac{g\sigma}{4\pi W} \int_0^{\infty} dk \cdot k J_0(rk) e^{-|z|k} \cdot \{d_1(k) \cos \sqrt{g\sigma k}\, t + d_2(k) \sin \sqrt{g\sigma k}\, t\} \quad . \quad (12)$$

(The principal value of the expression for ϕ_3 is understood). $\phi_1 + \phi_2$ represents the solution when gravity is absent, and the remaining terms $\phi_3 + \phi_4$ represent the effect of gravity which is sought.

From Eq. (10) it follows that

$$|F(m)| < C\, e^{b|\mathrm{im}(m)|}$$

where C is a fixed constant. It will also be assumed that the body is symmetrical about its centre so that $f(z) = f(-z)$ and $F(m) = F(-m)$. If the body is not in the interface, so that $|Wt| > b$, it is possible to integrate round an infinite semi-circle in the m-plane and so evaluate the m-integral which occurs in the integral expression for ϕ_3. The residues from the poles at $m = \pm ik$, $m = \pm \sqrt{g\sigma k}/W$ give a contribution to this integral, and we obtain

$$\phi_3 = -(\mathrm{sgn}\ Wt)\frac{g\sigma}{4W}\int_0^\infty dk\, \frac{J_0(rk)\, e^{-|z|k}\, F\left(\frac{\sqrt{g\sigma k}}{W}\right)}{k + g\sigma/W^2}\left\{\sigma k \cos\sqrt{g\sigma k}\, t - \frac{\sqrt{g\sigma k}}{W}\sin\sqrt{g\sigma k}\, t\right\}$$

$$-\frac{g\sigma}{4W}\int_0^\infty dk\, \frac{k\, J_0(rk)\, e^{-|z|k}\, F(ik)}{k + g\sigma/W^2}\cdot(1 - (\mathrm{sgn}\ Wt\, z)\,\sigma^2)\, e^{-|Wt|k}.$$

The positive root is taken throughout. The first integral is the contribution from the residues and represents a wave train on the interface. If no waves are present in the initial condition when $|Wt| > b$, $t < 0$, it follows that $d_1(k)$ and $d_2(k)$ are such that

$$\phi_4 = -(\mathrm{sgn}\ Wz)\frac{g\sigma}{4W}\int_0^\infty dk\, \frac{J_0(rk)\, e^{-|z|k}\, F\left(\frac{\sqrt{g\sigma k}}{W}\right)}{k + \frac{g\sigma}{W^2}}\cdot\left\{k\cos\sqrt{g\sigma k}\, t - \frac{\sqrt{g\sigma k}}{W}\sin\sqrt{g\sigma k}\, t,\right\}, \quad . \quad (13)$$

since then the residues from ϕ_3 cancel with ϕ_4 and all wave expressions such as (12) are absent. If we put $t > 0$, $|Wt| > b$, the potential representing the waves generated during the passage of the body through the interface is obtained from ϕ_4 plus the residues from ϕ_3. In this case the residues reinforce ϕ_4, and so

$$\phi_{\text{waves}} = 2\,\phi_4,$$

where ϕ_4 is given by expression (13). The velocity potential which represents the effect of gravity is given by $\phi_3 + \phi_4$ where ϕ_3 is given by expression (12) and ϕ_4 by expression (13). If the shape of the body is written as

$$r_a = a f_1\left(\frac{z}{b}\right) \text{ where } f_1\left(\frac{z}{b}\right) = 0 \text{ if } |z| > b,$$

then Eq. (10) may be written

$$F(m) = 2a^2 b\, F_1(mb)$$

where

$$F_1(m) = \int_0^1 d\theta\, \{f_1(\theta)\}^2 \cos m\theta.$$

If we substitute this expression for $F(m)$ in the expressions for ϕ_3 and ϕ_4, and write

$$\frac{Wt}{b} = \tau;\ \frac{z}{b} = z_1;\ \frac{r}{b} = r_1; \quad . \quad . \quad . \quad . \quad (14)$$

we obtain

$$\phi_3 = (\mathrm{sgn}\ z_1)\frac{a^2 W\nu}{2\pi b}\int_0^\infty dk \int_{-\infty}^\infty dm\, \frac{k\, J_0(r_1 k)\, m\, F_1(m)\,(i\sigma k - m)\, e^{im\tau - |z_1|k}}{(k^2 + m^2)(m^2 - \nu k)}, \quad (15)$$

and

$$\phi_4 = -(\operatorname{sgn} W z_1)\frac{a^2 W \nu}{2b}\int_0^\infty dk \frac{J_0(r_1 k)\, e^{-|z_1|k} F_1(\sqrt{\nu k})}{k+\nu}\{\sigma k \cos\sqrt{\nu k}\,\tau - \sqrt{\nu k}\sin\sqrt{\nu k}\,\tau\} \quad . \quad (16$$

From Eq. 13 we see that the amplitude of the wave system is given by

$$\frac{a^2}{b} \times \text{some function of } \nu \text{ and } \tau.$$

4. The wave energy

The upthrust R on the body, owing to the wave resistance, is found from a consideration of the pressure forces on its surface. Euler's equation of motion in the horizontal direction is

$$\rho\frac{\partial u}{\partial t} + \frac{\partial p}{\partial r} = 0$$

if second-order terms are omitted. Hence if the static thrust is omitted, the pressure p is given approximately by $p = \rho\,\partial\phi/\partial t$, and the upthrust R is given by

$$R = \int_{-\infty}^{\infty} dz\,\{p_a f'(z - Wt)\}\, 2\pi f(z - Wt)$$

where p_a denotes the pressure at the surface of the body. The wave resistance is found by the substitution for p_a in terms of ϕ_3 and ϕ_4:

$$R \simeq 2\pi\int_{-\infty}^{\infty} dz\, f(z - Wt) f'(z - Wt)\,\rho\frac{\partial}{\partial t}(\phi_3 + \phi_4),$$

where ϕ_3 and ϕ_4 are given by (12) and (13), with $r = r_a$. If we write

$$f(z - Wt) f'(z - Wt) = \frac{1}{4\pi i}\int_{-\infty}^{\infty} ds\, sF(s)\, e^{-is(z - Wt)},$$

we obtain

$$R = \frac{1}{2i}\left\{\rho_2\int_{-\infty}^{0} dz\, e^{-isz} + \rho_1\int_0^\infty dz\, e^{-isz}\right\}\left\{\int_{-\infty}^{\infty} ds\, sF(s)\, e^{isWt}\frac{\partial}{\partial t}(\phi_3 + \phi_4)\right\}.$$

We assume that $J_0(r_a k) \doteq 1$, and carry out the differentiation with respect to t and the integration with respect to z under the integral signs $\int ds$, $\int dk$ and $\int dm$. Expressions for R_3 and R_4 are then obtained, where R_3 corresponds to ϕ_3, R_4 corresponds to ϕ_4, and $R_3 + R_4 = R$. In terms of the notation (14) these expressions are:

$$R_3 = \frac{2a^4 W^2(\rho_1+\rho_2)\nu}{\pi b^2}$$

$$\int_0^\infty ds\int_0^\infty dk\int_{-\infty}^{\infty} dm\,\frac{F_1(s) F_1(m)\, skm^2(\sigma k\cos m\tau - m\sin m\tau)(s\cos s\tau + \sigma k\sin s\tau)}{(k^2+m^2)(k^2+s^2)(m^2-\nu k)}, \quad (17)$$

and

$$R_4 = -(\operatorname{sgn} W)\cdot\frac{a^4 W^2(\rho_1+\rho_2)\nu^{3/2}}{b^2}\int_0^\infty ds\int_0^\infty dk\,\frac{F_1(s) F_1(\sqrt{\nu k})\, s\sqrt{k}}{(k^2+s^2)(k+\nu)}\cdot$$

$$(\sigma k\sin\sqrt{\nu k}\,\tau + \sqrt{\nu k}\cos\sqrt{\nu k}\,t)(s\cos s\tau + \sigma k\sin s\tau) \quad . \quad (18)$$

If $|\tau| > 1$, the integration with respect to s yields a factor $k\,e^{-|\tau|k}$ and the integration

with respect to m yields a bounded integrand for the remaining integration with respect to k. Therefore

$$|R_{3,4}| < c \int_0^\infty dk \,.\, k^2 e^{-|\tau| k} = 0\,(\tau^{-3}).$$

It follows that the wave resistance to motion when the body is not crossing the interface is of the order (distance)$^{-3}$, confirming that most of the wave energy is developed as the body crosses the interface.

The work done by the body against the wave resistance is E, say, where

$$\begin{aligned} E &= -(\text{sgn } W) \int_{-\infty}^{\infty} dz\,(R_3 + R_4) \\ &= -(\text{sgn})\, W \,.\, 2b \,.\, \lim_{T\to\infty} \int_0^T d\tau\,(R_3^+ + R_4^+) \\ &= E_3 + E_4, \text{ say,} \end{aligned}$$

and where R_3^+ and R_4^+ denote those parts of expressions (17) and (18) which are even functions of τ. Now the integrand of R_3^+ contains the factor

$$(s+m)\cos(s+m)\,\tau - (s-m)\cos(s-m)\,\tau.$$

If the integration with respect to τ is carried out under the integral signs $\int ds$, $\int dk$ and $\int dm$, and the limit as $T \to \infty$ is taken, this factor becomes

$$\tfrac{1}{2}\{(s+m)\,\delta(s+m) + (s-m)\,\delta(s-m)\},$$

where δ denotes the Dirac delta function. The integration with respect to s then shows that E_3 is zero. To evaluate E_4, we again integrate with respect to τ under the integral signs, and obtain

$$E_4 = \frac{2a^4 W^2 (\rho_1+\rho_2)\nu^{3/2}}{b^2} \int_0^\infty ds \int_0^\infty dk \frac{F_1(s)\,F_1(\sqrt{\nu k})\,s\sqrt{k}}{(k+\nu)(k^2+s^2)} \lim_{T\to\infty} \int_0^T d\tau\,(\sigma^2 k \sin s\tau \sin\sqrt{\nu k}\,\tau + s\sqrt{\nu k}\cos s\tau \cos\sqrt{\nu k}\,\tau).$$

If we write $k = \lambda^2/\nu$, integrate with respect to τ, and take the limit as $T \to \infty$, this becomes

$$\begin{aligned} E_4 &= \frac{2a^4 W^2 (\rho_1+\rho_2)\nu}{b} \int_0^\infty ds \int_0^\infty d\lambda \frac{F_1(s)\,F_1(\lambda)\,\lambda^3}{(\lambda^2+s^2)(\lambda^4+s^2\nu^2)} \{(\nu^2 s - \sigma^2\lambda^3)\,\delta(s+\lambda) + (\nu^2 s + \sigma^2\lambda^3)\,\delta(s-\lambda)\} \\ &= 2a^4 g\,\Delta\rho \int_0^\infty ds \frac{s^3\,[F_1(s)]^2}{(s^2+\nu^2)^2}(\sigma^2 s^2 + \nu^2). \end{aligned}$$

This result may be written

$$\frac{E}{a^4 g\,\Delta\rho} = (1-\sigma^2)\,e_1(\nu) + \sigma^2 e_2(\nu), \quad . \quad . \quad . \quad (19)$$

where

$$e_1(\nu) = 2\nu^2 \int_0^\infty ds \,.\, \frac{s^3\,[F_1(s)]^2}{(\nu^2+s^2)^2}, \quad . \quad . \quad . \quad . \quad (20)$$

and

$$e_2(\nu) = 2\int_0^\infty ds \,.\, \frac{s^3\,[(F_1(s)]^2}{\nu^2+s^2} \quad . \quad . \quad . \quad . \quad (21)$$

The expression (19) may also be derived directly from a consideration of the wave energy from ϕ_4, Eq. (16). Approximate methods were used to evaluate the integral expressions for $e_1(\nu)$ and $e_2(\nu)$. If we write $e_1(\nu)$ in the form

$$e_1(\nu) = -\nu \int_0^\infty ds \,.\, \tan^{-1}\frac{s}{\nu} \cdot \frac{d^2}{ds^2}\{s\,F_1(s)\}^2$$

and approximate to $\tan^{-1} s/\nu$ by the approximation

$$\tan^{-1}\frac{s}{\nu} \simeq \begin{cases} \dfrac{\pi s}{4\nu}, & 0 < s < 2\nu \\ \pi/2, & 2\nu < s, \end{cases}$$

we obtain as a rough approximation

$$e_1(\nu) \simeq \pi\nu^2 \{F_1(2\nu)\}^2, \qquad (20a)$$

for small or moderate values of ν (compared with 1). If ν is large,

$$e_1(\nu) \simeq \frac{2}{\nu^2}\int_0^\infty ds \,.\, s^3 \{F_1(s)\}^2 \,. \qquad (20b)$$

Similar methods for the integral for $e_2(\nu)$ give

$$e_2(\nu) \simeq \frac{\pi}{2\nu^2}\int_0^{2\nu} ds \,.\, s^3 \{F_1(s)\}^2,$$

if ν is small or moderate (compared with 1), and

$$e_2(\nu) \simeq \frac{2}{\nu^2}\int_0^\infty ds \,.\, s^3 \{F_1(s)\}^2,$$

if ν is large. The curves of $e_1(\nu)$ and $e_2(\nu)$ for the spheroid and spindle (cosine) shaped body are shown in Figs. 2 and 3. In these figures a modification of variable $\nu \to \pi\nu_1$ has been made for convenience of calculation.

5. The two-dimensional case

An exactly similar analysis as used in Sections 2 to 4 may be used to estimate the energy needed to overcome the wave resistance at the interface. In this case the body is supposed to be an infinite cylinder whose axis is horizontal and whose slender cross section $r_a = f(z)$ is symmetrical about vertical and horizontal planes drawn through its centre. In the following the same notation is used as in Sections 2 to 4. The boundary condition which corresponds to Eq. (3) is

$$u_{r=0} = -\left(\frac{\partial\phi}{\partial r}\right)_{r=0} = -Wf'(z - Wt) \,. \qquad (3)'$$

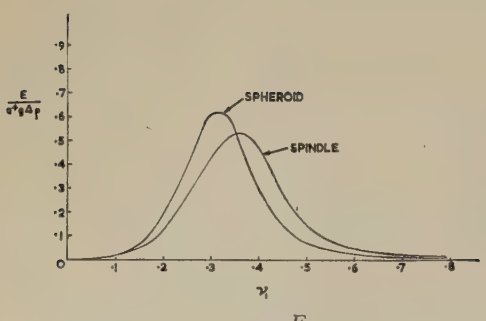

Figure 2. Curves of $\dfrac{E}{a^4 g \Delta\rho}$ against ν_1.

Case 1 : Small density difference.

$$\nu_1 = \frac{1}{\pi}\sqrt{\frac{gb(\rho_2 - \rho_1)}{W(\rho_2 + \rho_1)}} = \frac{1}{\pi}\sqrt{\frac{gb\,\Delta\rho}{2W\rho}}$$

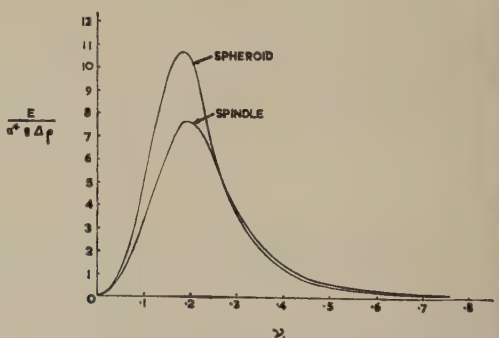

Figure 3. Curves of $\dfrac{E}{a^4 g \Delta\rho}$ against ν_1.

Case 2 : Upper fluid absent.

$$\nu_1 = \frac{1}{\pi}\sqrt{\frac{gb(\rho_2 - \rho_1)}{W(\rho_2 + \rho_1)}} = \frac{1}{\pi}\sqrt{\frac{gb}{W}}$$

The Laplace equation for the velocity potential is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \qquad (7)'$$

and we write

$$\bar{\phi}(r, k, t) = \int_0^\infty dr \cos kr \,.\, \phi(r, z, t) \qquad (8)'$$

If we multiply (7)′ by $\cos kr$ and integrate with respect to r from 0 to ∞, we obtain

$$-k^2 \bar{\phi} + \frac{\partial^2 \bar{\phi}}{\partial z^2} = W f'(z - Wt) \qquad (9)'$$

Here we assume that $r.\ \phi(r, z, t) \to 0$ as $r \to 0$, and that $\phi \to 0$ as $r \to \infty$. The r.h.s. of this equation is replaced by

$$\frac{W}{2\pi i} m F(m) e^{-im(z - Wt)}, \qquad (10)'$$

where

$$F(m) = \int_{-\infty}^{\infty} dz \,.\, f(z) e^{imz} \qquad (10)'$$

Eq. (10)′ may be written

$$F(m) = 2ab F_1(mb),$$

where $F_1(m) = \int_0^1 d\theta f_1(\theta) \cos m\theta$, and where the cross-section of the cylinder is given by

$$r_a = a f_1\left(\frac{z}{b}\right), \quad \left|\frac{z}{b}\right| < 1$$

$$= 0, \quad \left|\frac{z}{b}\right| > 1.$$

An analysis exactly similar to that used previously gives

$$\phi_{\text{waves}} = 2\phi_4,$$

where now

$$\phi_4 = -(\operatorname{sgn} W z_1)\, a W \nu \int_0^\infty dk \frac{\cos(r_1 k)\, e^{-|z_1|k} F_1(\sqrt{\nu k})}{k(\nu + k)} \,.\, (\sigma k \cos\sqrt{\nu k}\,\tau - \sqrt{\nu k} \sin\sqrt{\nu k}\,\tau) \qquad (16)'$$

Expressions for the wave resistance and therefore for the energy E per unit length of the cylinder may be found as before. Then $p \simeq \rho\, \partial\phi/\partial t$ and

$$R = 2 \int_{-\infty}^{\infty} dz\, p_a \,.\, f'(z - Wt).$$

If similar linear approximations are made as before, then it follows that

$$\frac{E}{a^4 bg \Delta\rho} = (1 - \sigma^2)\, \eta_1(\nu) + \sigma^2 \eta_2(\nu) \qquad (19)'$$

where

$$\eta_1(\nu) = \frac{8\nu^3}{\pi} \int_0^\infty ds \frac{s\{F_1(s)\}^2}{(s^2 + \nu^2)^2}, \qquad (20)'$$

and

$$\eta_2(\nu) = \frac{8\nu}{\pi} \int_0^\infty ds \,.\, \frac{s\{F_1(s)\}^2}{s^2 + \nu^2} \qquad (21)'$$

A similar type of approximation to that used to obtain an approximation to $e_1(\nu)$ and $e_2(\nu)$ gives in these cases:

$$\eta_1(\nu) \simeq \frac{8}{\pi\nu}\int_0^\infty ds\,.\,s\{F_1(s)\}^2,$$

and

$$\eta_2(\nu) \simeq \frac{8}{\pi\nu}\int_0^\infty ds\,.\,s\{F_1(s)\}^2$$

if ν is large, and

$$\eta_1(\nu) \simeq \nu\{F_1(2\nu)\}^2,$$

and

$$\eta_2(\nu) \simeq \frac{2}{\nu}\cdot\int_0^{2\nu} ds\,.\,s\{F_1(s)\}^2$$

if ν is small or moderate, (compared with 1).

6. Applications and conclusions

For the thermal in ascent, Scorer's value for the velocity, Scorer (1952) is

$$W = \sqrt{gBa} \qquad . \quad . \quad . \quad . \quad . \quad (22)$$

where $2a$ is the width of the thermal, and B is $\delta\rho/\rho$, where $\delta\rho$ is the density deficit of the thermal. We assume that for this purpose the thermal is a sphere whose diameter is equal to the width of the thermal. As mentioned in the Introduction, the case needed for the thermal is that of a small density difference between the fluids, a typical value of $\Delta\rho$ for an inversion being 2 per cent. Hence in (19), σ^2 may be neglected, and the wave energy developed $= a^4 g\Delta\rho\, e_1(\sqrt{\sigma gb}/W)$

$$= a^4 g\,\Delta\rho\, e_1\left(\sqrt{\frac{\Delta\rho}{2\delta\rho}}\right)$$

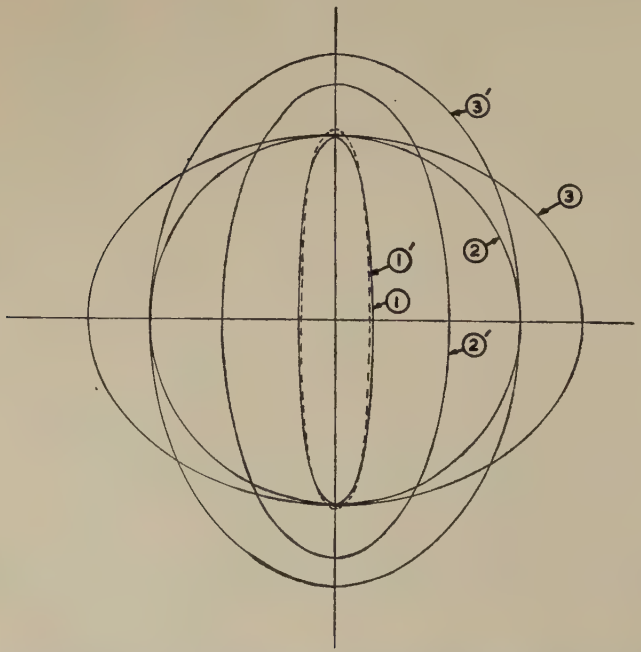

Figure 4. Accuracy of the slender-body approximation. The spheroids ①′, ②′, and ③′ are the slender-body approximations to the spheroids ①, ② and ③ respectively. ② is a sphere.

by use of (22). Some comparison for this wave energy is required. The internal kinetic energy of the thermal will be at least as great as the kinetic energy of the thermal moving as a solid homogeneous sphere. To this energy is to be added the kinetic energy of the surroundings. This is accounted for by the virtual mass coefficient, which for the sphere is $\frac{1}{2}$. Hence the kinetic energy of translation is at least $\frac{3}{4} \cdot \frac{4}{3} \pi a^3 W^2 (\rho_2 - \delta\rho)$

$$\simeq \pi a^4 g \, \delta\rho. \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad (23)$$

The effect of the change in kinetic energy of the surrounding fluid as the thermal moves to less dense surroundings is neglected. The work done by the buoyancy forces as the sphere crosses the interface is to be taken into account. (The work done by the buoyancy forces when the sphere is not crossing the interface is not relevant since most of the wave energy is developed during the time of crossing). This work is easily shown to be

$$2a \left(\tfrac{4}{3} \pi a^3\right) g \left\{\frac{\rho_2 + \rho_1}{2} - (\rho_2 - \delta\rho)\right\}$$

$$= \frac{8\pi a^4}{3} \cdot g \, \delta\rho \left(1 - \frac{\Delta\rho}{2\delta\rho}\right) = \frac{8\pi a^4}{3} g \, \delta\rho \, . \, (1 - \nu_0), \qquad . \qquad . \qquad (24)$$

where $\nu_0 = \dfrac{\Delta\rho}{2\delta\rho}$.

The wave energy is $a^4 g \, \Delta\rho \, e_1 (\nu_0)$, and this is to be compared with expressions (23) and (24). Thus.

$$\frac{\text{wave energy}}{\text{energy of translation}} = \frac{2}{\pi} \cdot \nu_0 e_1 (\nu_0), \qquad . \qquad . \qquad . \qquad (25)$$

and

$$\frac{\text{wave energy}}{\text{work done by the buoyancy}} = \frac{3}{4\pi} \cdot \frac{\nu_0 \, . \, e_1 (\nu_0)}{1 - \nu_0} \qquad . \qquad . \qquad (26)$$

The curve of $e_1 (\nu)$ for a sphere shows that a maximum occurs at $\nu = 1$ approximately, and that $e_1 (\nu)$ is then about 0·64. Hence when $\Delta\rho = 2\delta\rho$, the work done by the buoyancy is zero, and the ratio of the wave energy to the kinetic energy of motion is about 0·4. On the other hand, if $\Delta\rho = \delta\rho$, the ratios (25) and (26) become about ·03 and ·02 respectively. Typical values for the atmosphere are

$$\Delta\rho = 2 \text{ per cent}, \quad \delta\rho = 5 \text{ per cent},$$

and the ratios (25) and (26) are then ·0024 and ·0009 approximately. Thus the energy lost by the generation of waves as the sphere crosses the interface will substantially affect the ensuing motion only in cases where $\Delta\rho \simeq 2\delta\rho$; that is, in cases where the average density of the two fluids is equal to the density of the sphere. Only in the case where the buoyancy changes sign as the thermal passes through the inversion will the energy spent in making waves have a significant effect on the motion: in this case the change in buoyancy will subsequently completely alter the character of the motion, and hence the thermal itself. If the inversion is such that the change in buoyancy as the thermal penetrates it will not radically change the motion of the thermal, the energy dispersed into wave motions of the thin inversion layer may be neglected.

The curves of $e_1 (\nu)$ and $e_2 (\nu)$ agree qualitatively with what might be expected from physical arguments. At high speeds and low speeds the wave energy is small, for at high speeds there is little time for the waves to develop, and at low speeds there is little or no deformation of the interface outside the region of penetration. At intermediate speeds, however, the front of the body will generate a wave crest which will subside into a trough at the same time as the rear of the body leaves the interface. The trough then subsides still further. Thus there is a condition which is most favourable to the generation of

waves. From expressions (20a) and (20b) it follows that for small ν, $e_1(\nu) \propto \nu^2$ and for large ν, $e_1(\nu) \propto \nu^{-2}$, so there is an optimum condition for the maximum generation of waves for any body. This condition occurs at some intermediate speed given by $W^2 = n\,bg\Delta\rho/2\rho$, say, where n is some number which depends only on the shape of the body. For the thermal, this criterion becomes $\nu_0 = n$, on the use of expression (22).

Similar remarks for the case of bent-over, buoyant, chimney plume apply as in the case of the thermal: the plume is regarded as a horizontal cylinder which rises through an inversion, and this case corresponds to the symmetrical two-dimensional case. The outcome is that the wave-making process as the plume passes through an inversion may be neglected if the buoyancy change does not markedly affect the subsequent motion of the plume.

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Ice accretion and turbulence on North Atlantic air routes

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Summary

Routine observations made in transport aircraft in January and July, 1955-57, are analysed to obtain the frequency of ice accretion in relation to air temperature and of turbulence in relation to wind speed and cloud. Comparative freedom from icing in the range 0° to − 3°C is attributed to the effects of kinetic heating. In January the frequency attains a maximum of 43 per cent of occasions in cloud at − 4° to − 7°C; in July 20 per cent at − 4° to − 11°C. There is a secondary maximum in both months near the lowest temperatures at which icing is reported, which is attributed to flight through the upper parts of convection cloud. The frequency of icing in July is generally much less than at the same temperatures in January.

Turbulence is reported on about 5 per cent of all occasions in January and on 2 per cent in July; in cloud the frequencies are 18 and 12 per cent respectively. The frequency of turbulence tends to increase with the wind speed.

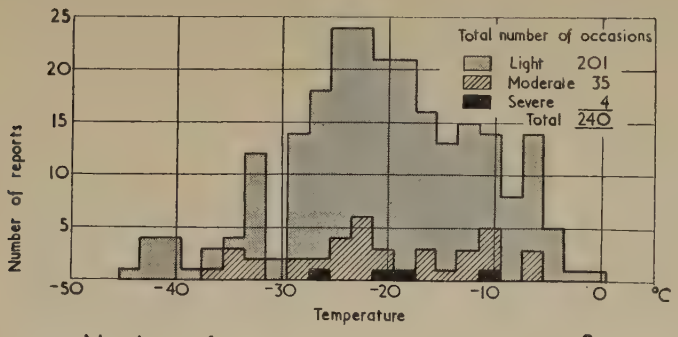
1. Introduction

Transport aircraft on long routes are required to make position and meteorological reports at regular intervals. The reports are received at certain air terminals where they provide the aviation forecasters with current information on conditions along the routes; moreover, any reports of conditions likely to affect the safety of aircraft are re-transmitted for the benefit of other aircraft and other forecast offices. With the passage of time, large numbers of reports accumulate. It is the object of this paper to show that a statistical examination of these reports can lead to an improvement of our knowledge on at least two aspects of meteorology and the operation of aircraft, namely ice accretion and turbulence. The reports used in this paper are the POMAR and AIREP reports received at London Airport from transport aircraft on North Atlantic routes in January and July, 1955-57. The contents of these reports were first tabulated in manuscript and then entered on Hollerith cards to facilitate sorting.

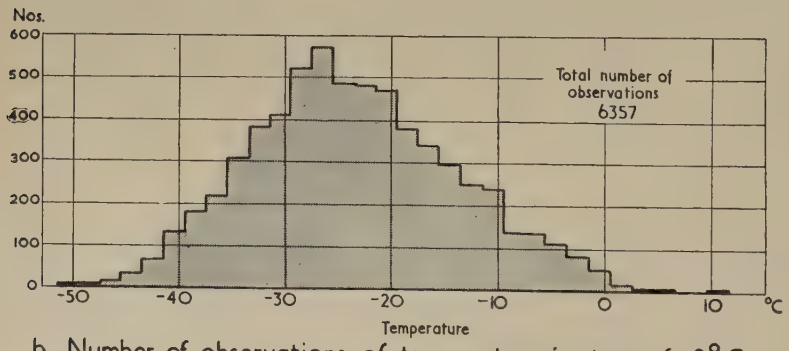
2. Ice accretion on aircraft

The observations of aircraft icing are classified subjectively as light, moderate or severe. No information is available from the transmitted reports regarding the appearance of the ice or the parts of the aircraft affected, but airframe icing can reasonably be assumed in all cases, in contrast with engine icing which is very unusual in the circumstances. The inclusion of icing in the routine meteorological reports is obligatory when it occurs. However, the planning of flights to avoid regions of potential icing, the taking of avoiding action in flight, and the use of de-icing equipment are liable to affect both the number and intensity of the reported occurrences. This means that although the reports of icing may produce satisfactory statistics as regards the operation of aircraft, they probably underestimate the frequency and intensity of potential icing regions on the air routes. Hitherto most statistical studies of ice accretion on aircraft have been based on mainly positive occurrences only. Thus Jones (1959) remarks on the ' natural tendency to report only those occurrences which were exceptional in some way and reports of no ice accretion are rarely received.' This particular difficulty is overcome in the present paper by the use of routine meteorological reports, made at regular intervals throughout a flight, whether or not icing happens to be occurring on the routes concerned.

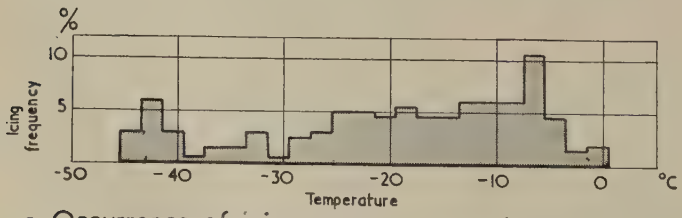
The occurrence of ice accretion will be discussed in relation to ambient temperature. The temperature is reported only when the aircraft is in steady level flight, and it is corrected for instrumental error and air-speed. With the many types of aircraft concerned, and a number of operating companies, there are no doubt several kinds of thermometer and a variety of exposures in use at any one time. A comparison was made between the aircraft temperatures and those obtained by radiosonde from the Weather Ship *J* stationed at lat. 52½°N, long. 20°W. The aircraft observations used for this purpose were those made within an area centred at the nominal position of the weather ship and extending over 5 degrees of latitude and 10 degrees of longitude; this area is a little larger than the area over which the weather ship is allowed to operate while still being regarded as on station. The comparison was made for both January and July for the years 1955-57 at several heights from 6,000 to 25,000 ft. No significant difference was found in either the mean temperatures or the standard deviations, except that in July the means derived from the aircraft observations were systematically higher than those from the Weather Ship by 0·6°C. With this slight difference, it appears therefore that the temperatures recorded on the aircraft can be regarded as statistically reliable.



a. Number of reports of airframe icing in 2°C steps



b. Number of observations of temperature in steps of 2°C.



c. Occurrence of icing as percentage of number of observations of temperature in 2°C. steps.

Figure 1. Distribution of airframe icing in relation to temperature, North Atlantic, January, 1955-57.

TABLE 1. AIRFRAME ICING IN RELATION TO TEMPERATURE, JANUARY, 1955-57

Temperature °C	⩾ 1	0 −1	−2 −3	−4 −5	−6 −7	−8 −9	−10 −11	−12 −13	−14 −15	−16 −17	−18 −19	−20 −21	−22 −23	−24 −25	−26 −27	−28 −29	−30 −31	−32 −33	−34 −35	−36 −37	−38 −39	−40 −41	−42 −43	−44 −45	⩽ −46	Total
No. of observations	33	50	79	110	134	134	240	252	297	342	382	472	481	486	569	520	412	381	309	221	180	134	69	36	34	6,357
Icing																										
Light		1	1	5	11	8	9	12	12	13	20	18	18	20	16	12	2	10	1	2	1	4	4	1		201
Moderate					3		4	3	1	3		2	6	4	1	2		2	3	1						35
Severe							1				1	1			1											4
Total		1	1	5	14	8	14	15	13	16	21	21	24	24	18	14	2	12	4	3	1	4	4	1		240
Percentage		2·0	1·3	4·5	10·4	6·0	5·8	6·0	4·4	4·7	5·5	4·4	5·0	4·9	3·2	2·7	0·5	3·1	1·3	1·4	0·6	3·0	5·8	2·8		3·8

(a) *January*

Details of the observations of icing and of temperature are presented in Table 1 and Fig. 1. Out of 6,357 observations of temperature 6,324 were at or below 0°C and there were 240 reports of icing. The distribution of these occasions with temperature (Fig. 1 (a)) is largely conditioned by the heights or temperatures at which the aircraft were operating; with few exceptions, the heights ranged from 6,000 to 25,000 ft with a maximum at 19,000 ft, corresponding with − 23°C in the International Standard Atmosphere. The frequency distribution of observed temperature is shown in Fig. 1 (b), while Fig. 1 (c) shows the frequency of icing as a percentage of the total number of observations in each 2° step of temperature. The aircraft reports usually include information on 'flying conditions,' by which is meant the situation of the aircraft with regard to cloud. The occurrence of icing with respect to flying conditions is shown in Table 2 for all those occasions when this information is available.

TABLE 2. Observations of icing in relation to flying conditions, January, 1955-57

	Flying conditions						
Icing	No cloud	Above cloud	Below cloud	Between layers	In and out of cloud	In cloud	Total
Light	–	–	12	9	75	86	182
Moderate	–	–	5	1	12	14	32
Severe	–	–	1	1	2	–	4
Total	–	–	18	11	89	100	218

It is seen that 189 occasions (87 per cent) of icing occur when the aircraft is continuously or intermittently in cloud, and the remaining 29 occasions (13 per cent) when the aircraft is in any case flying below cloud, are probably associated with precipitation although it is not possible to confirm this from the reports. The 189 reports of ice accretion when flying in cloud (including 'in and out of cloud') are distributed in regard to temperature as shown in Table 3, while Fig. 3 shows the percentage frequency of icing in cloud in each 4°C step of temperature.

TABLE 3. Icing in cloud in relation to temperature, January, 1955-57

Temperature (°C)	⩾1	0 to −3	−4 to −7	−8 to −11	−12 to −15	−16 to −19	−20 to −23	−24 to −27	−28 to −31	−32 to −35	−36 to −39	−40 to −43	−44 to −47	⩽−48	Total ⩽0
Number of observations															
In cloud	1	20	35	52	94	104	157	159	109	81	40	33	6	2	892
Icing	0	2	15	15	24	24	41	31	14	13	3	6	1	0	189
Icing percentage	0	10	43	29	26	23	26	19	13	16	7	18	17	0	21

These tables and histograms have several points of interest. In the range 0° to − 3°C there are only two cases of icing out of 20 observations. The small risk here can probably be attributed to the effects of kinetic heating which (at least in clear air) is given approximately by $(V/100)^2$ °C where V is true airspeed in knots (Meteorological Office 1945) and so amounts to about 4°C for airspeeds of about 200 kt. In regard to the frequency of icing in cloud, Table 3 and Fig. 3 show a pronounced maximum of 43 per cent in

TABLE 4. AIRFRAME ICING IN RELATION TO TEMPERATURE, JULY, 1955-57

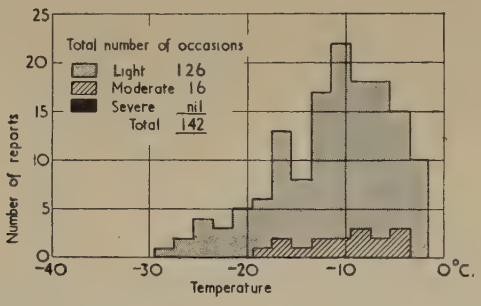
Temperature °C	0 −1	−2 −3	−4 −5	−6 −7	−8 −9	−10 −11	−12 −13	−14 −15	−16 −17	−18 −19	−20 −21	−22 −23	−24 −25	−26 −27	−28 −29	−30 −31	−32 −33	−34 −35	−36 −37	−38 −39	⩽−40	Tota
No. of observations																						
Temperature	507	655	674	655	721	750	703	736	639	528	517	452	260	205	100	74	23	18	1	1	3	8,222
Icing																						
Light		10	12	16	15	20	15	7	11	5	5	3	4	2	1							126
Moderate			3	2	3	2	2	1	2	1												16
Severe																						–
Total		10	15	18	18	22	17	8	13	6	5	3	4	2	1							142
Percentage		1·5	2·2	2·7	2·5	2·9	2·4	1·1	2·0	1·1	1·0	0·7	1·5	1·0	1·0							1·7

the step — 4° to — 7°C, with the frequency falling off rather irregularly at lower tempera tures down to a minimum of 7 per cent at — 36° to — 39°C, followed by a further ris to 18 per cent in the range — 40° to — 47°C (or 20 per cent in the range — 40° to — 44°C the latter being the lowest temperature at which icing was reported).

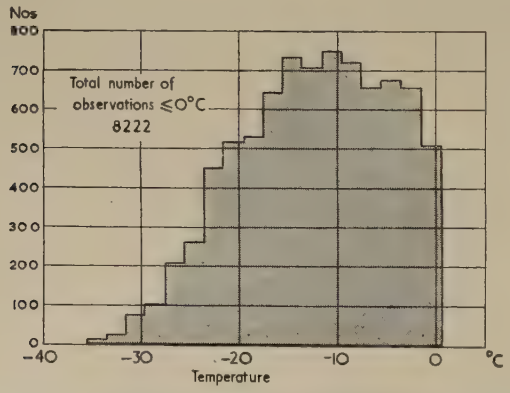
The occurrence of 9 cases of icing at reported temperatures — 40° to — 44°C (Table 1 appears at first sight to conflict with experimental evidence that supercooled droplet freeze spontaneously at about — 40°C (Mason 1957). From time to time there are report of icing on aircraft at much lower temperatures (e.g., Shaw 1954) which may perhap be explained by Mason's remark that although some water-drop clouds appear to *form* a temperatures considerably below — 40°C, the droplets would not be expected to *persis* in the supercooled state at these temperatures. The 9 cases mentioned are made up a follows :

Temperature °C	— 40	— 41	— 42	— 43	— 44
Reports of icing	3	1	3	1	1

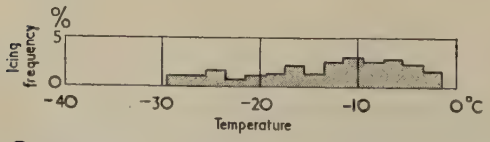
The critical temperature for spontaneous freezing, as determined by laboratory experi ments, is not precise, estimates varying by a degree or so from — 40°C according to the laboratory conditions. If in addition there is an uncertainty of about 2°C in the aircraf



a Number of reports of airframe icing in 2°C steps



b Number of observations of temperature in steps of 2°C.



c. Occurrence of icing as percentage of number of observations of temperature in 2°C. steps.

Figure 2. Distribution of airframe icing in relation to temperature, North Atlantic, July, 1955-57.

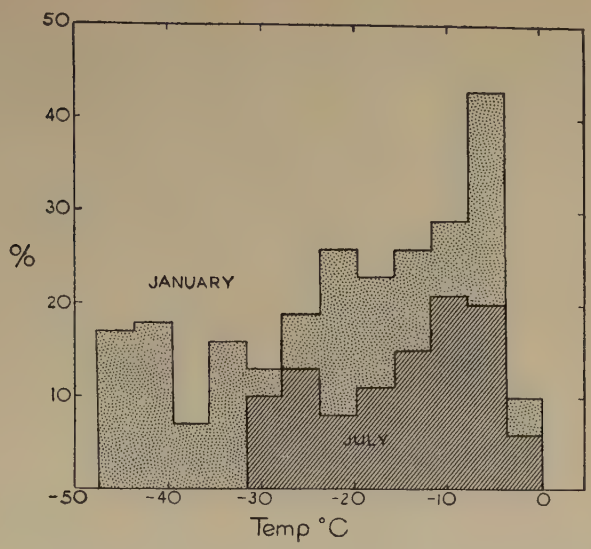


Figure 3. Percentage frequency of icing in cloud.

emperatures, then the present observations are not necessarily inconsistent with experimental results. The increased frequency of icing in cloud at these temperatures will be liscussed with the July results.

(*b*) *July*

A similar analysis was carried out for July for the same years, 1955-57, and is shown n Tables 4-6 and Figs. 2 and 3. The distribution of aircraft heights is similar to that in anuary, but the aircraft reports are more numerous by nearly 40 per cent. In consequence f the higher summer temperatures, there are few recordings below — 35°C, while the owest temperature at which icing was reported is — 29°C.

The frequency of icing in July is very much less than in January; the percentage of ll reports at or below 0°C is 1·7 in July against 3·8 in January, and the corresponding ercentages of icing in cloud are 14 and 21. This difference can probably be attributed o the greater frequency of convection cloud over the North Atlantic in winter compared ith summer, this being the cloud type most conducive to the icing of aircraft. Not nly is the icing less frequent in July, it is also, on the whole, less intense (see Table 4), nd there are no reports of severe icing. The distribution of icing with flying conditions s given in Table 5 for the 120 occasions when this information is available. The roportion of reports of icing in cloud (including ' in and out ') and below cloud (including 'between layers') are almost identical with those for January.

TABLE 5. Icing in relation to flying conditions, July, 1955-57

	Flying conditions						
Icing	No cloud	Above cloud	Below cloud	Between layers	In and out of cloud	In cloud	Total
Light	–	–	2	9	36	59	106
Moderate	–	–	2	1	4	7	14
evere	–	–	–	–	–	–	–
Total	–	–	4	10	40	66	120

Of the 106 occurrences of icing ' in ' or ' in and out ' of cloud, temperature observations are given for 104 which are distributed as shown in Table 6. In the range 0° to − 3°C the effects of kinetic heating are again evident. The frequency of icing in cloud is maintained close to 20 per cent from − 4° to − 11°C, falls to a minimum of 8 per cent at − 20° to − 23°C and then has a secondary maximum of 13 per cent at − 24° to − 27°C.

TABLE 6. ICING IN CLOUD IN RELATION TO TEMPERATURE, JULY, 1955-57

Temperature (°C)	⩾1	0 to −3	−4 to −7	−8 to −11	−12 to −15	−16 to −19	−20 to −23	−24 to −27	−28 to −31	−32 to −35	−36 to −39	⩽−40	Totals ⩽0
Number of observations													
In cloud	80	129	126	133	139	103	76	30	10	2	0	1	749
Icing	0	8	25	28	21	11	6	4	1	0	0	0	104
Icing percentage	0	6	20	21	15	11	8	13	10	0	–	0	14

The frequency distribution of icing in cloud in both January and July has a primary maximum at 4° or more below zero and a secondary maximum at about the lowest temperatures at which icing is reported. Although the temperatures at the secondary maximum are lower in January than in July, the occurrences take place in both months within the same range of aircraft heights, namely 18,000 to 24,000 ft. There are 12 reports of icing concerned (7 reports in January from − 44° to − 47°, and 5 reports in July from − 24° to − 31°); the corresponding flying conditions are reported on 9 occasions as ' in and out of cloud,' on 2 occasions as ' in cloud,' with the remaining case ' between layers.' The cloud type is reported on only 9 of these occasions, of which 6 are given as cumuliform. It appears, therefore, that most of these cases are associated with flight through the upper parts of cumulus or cumulonimbus clouds. A similar feature is evident in an analysis of reports of icing in relation to cloud types given by Jones (1956). All seasons are combined in his Table 3, where it is seen that out of over 600 positive reports of icing, there were none in the range − 36° to − 40°C, but four at temperatures of − 41°C or below, the lower limit not being specified; of these last, two were reported in cumulus or cumulonimbus, one in cirrus, and one out of cloud. Both the present observations and those discussed by Jones therefore suggest that the icing risk in cumulus and cumulonimbus has a secondary maximum in the upper part of the cloud, possibly even at temperatures below − 40°C. On the other hand it is possible that this result, like that of the supposed kinetic heating effect, is influenced by the way in which de-icing equipment is used; for example, if the equipment were used down to, but not below, a temperature of − 40°C, then that could explain an apparent increase in the frequency of icing at temperatures below − 40°C.

3. TURBULENCE

(*a*) *Distribution of turbulence with height*

When routine meteorological reports are made, the inclusion of turbulence is obligatory when it occurs, the intensity being estimated subjectively. From Table 7 it is seen that turbulence on the North Atlantic routes occurs on nearly 5 per cent of all occasions in January, but on only 2 per cent in July; moreover the intensity tends to be greater in January. Evidence is presented below that this seasonal variation is associated with differences in wind speed and in frequency of flight in cloud.

In the height range 6,000 to 9,000 ft, turbulence is equally frequent in both seasons. In the higher ranges it is much more frequent in January, with a pronounced maximum in that month of 8·5 per cent at 10,000 to 13,000 ft. Thereafter in both seasons the percentage frequency decreases as the height increases.

TABLE 7. DISTRIBUTION OF TURBULENCE WITH HEIGHT

Height ft	January All obs.	Turbulence Light	Mod.	Severe	Total	per cent	July All obs.	Turbulence Light	Mod.	Severe	Total	per cent
6- 9,000	374	8	6	–	14	3·7	846	22	8	–	30	3·6
0-13,000	940	58	22	–	80	8·5	1,993	57	16	1	74	3·7
4-17,000	2,014	69	17	2	88	4·4	2,908	50	6	1	57	2·0
8-21,000	2,694	92	20	1	113	4·2	3,232	37	3	–	40	1·2
2-25,000	606	11	7	1	19	3·1	731	7	–	–	7	1·0
Total	6,628	238	72	4	314	4·7	9,710	173	33	2	208	2·1

(*b*) *Turbulence and flying conditions*

From the flying conditions, which are reported on about 80 per cent of occasions, he frequency of occurrence of turbulence has been obtained for flight out of cloud and or flight in cloud, the latter also including cases reported as ' in and out of cloud.' The esults, which are presented in Table 8, emphasize in both months the frequently urbulent nature of flight in cloud in contrast with the rarely turbulent conditions out f cloud.

It is seen that flight in cloud occurs on 18 per cent of all occasions in January and on 2 per cent in July. Moreover the clouds tend to be more turbulent in January (19 per ent of occasions with turbulence) than in July (15 per cent); this would be expected from he more convective nature of the clouds in January, but it is perhaps influenced also by he stronger winds in January (see Table 9) in the same way that flight out of cloud is nore often turbulent in that month than in July.

TABLE 8. FREQUENCY OF TURBULENCE, (a) OUT OF CLOUD, and (b) IN CLOUD

Height ft	January a All	Turb.	per cent	b All	Turb.	per cent	July a All	Turb.	per cent	b All	Turb.	per cent
6- 9,000	183	3	2	82	9	11	544	9	2	92	15	16
0-13,000	607	21	3	171	53	31	1,336	11	1	236	57	24
4-17,000	1,349	30	2	254	55	22	1,842	16	1	215	29	13
8-21,000	1,882	45	2	382	62	16	2,203	12	0·5	277	23	8
2-25,000	382	12	3	72	6	8	452	1	0·2	59	4	7
Total	4,403	111	2·5	961	185	19	6,377	49	0·8	879	128	15

(*c*) *Turbulence and wind speed*

Reports of turbulence were analysed for all cases in which wind speed was reported, ith the results shown in Table 9. The wind is determined on the aircraft from successive xes, so that it is an average over a leg which may be of an hour's duration or extend over 0 degrees of longitude. These winds have no great individual accuracy, but used statis- cally they can lead to useful results. However, greater significance would be obtained ' spot ' winds were available by the use, for example, of a Doppler technique.

The outstanding feature of Table 9 is the tendency for the percentage frequency of irbulence to increase with the wind speed. Also in each speed group below 100 kt he winds are more often turbulent in January than they are in July, while at speeds greater nan 100 kt the frequencies are about the same in both months.

TABLE 9. TURBULENCE IN RELATION TO WIND SPEED

Speed (kt)	0-19	20-39	40-59	60-79	80-99	100-119	⩾ 120	Total
				January				
All obs.	683	1,757	1,499	825	441	188	108	5,501
Turbulence	21	68	86	55	34	10	10	284
Per cent	3	4	6	7	8	5	9	5
				July				
All obs.	1,456	3,585	2,233	598	123	22	10	7,957
Turbulence	25	84	42	16	8	1	1	176
Per cent	2	2	2	3	7	5	10	2

Another way of showing the connexion between turbulence and wind speed is to compare the mean wind on turbulent occasions with the mean wind on all occasions. In January the figures are 56 kt against 48 kt, in July they are 39 kt against 36 kt. Bannon (1951) obtained similar results, except that his observations concerned clear-air turbulence at heights from 20,000 to 40,000 ft. Since most of the present observations of turbulence are associated with flight in cloud (62 per cent in January and 72 per cent in July (see Table 8)) and cover heights from 6,000 to 25,000 ft, it follows that the tendency for turbulence to be associated with wind speed above normal holds at all heights from 6,000 to 40,000 ft, both in cloud and in clear air.

ACKNOWLEDGMENT

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Layer cloud over Southern England

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SUMMARY

599 morning weather reconnaissance flights from the Royal Aircraft Establishment, Farnborough, over a period of approximately four years have been examined to determine the frequency of occurrence and thickness of layer cloud over Southern England. The chance of encountering 4/8 or more layer cloud in any kilometre height range decreased with height from about 1 in 2 near the surface to about 1 in 50 at a height of 12 km. Cloud layers when encountered were more likely to exceed one or two kilometres depth in the upper troposphere than in the lower but the frequency of encountering a layer more than $1\frac{1}{2}$ km deep was about 7 to 8 per cent at all heights. About 1 in 9 of all flights encountered cloud above the melting level more than 3 km thick. An examination of clouds in the temperature band 0 to − 15°C suggested that conditions suitable for severe clear ice formation on some aircraft might be encountered on about one per cent of flights. The same examination suggests that seeding of layer clouds by silver-iodide smokes should produce some detectable modification of the cloud sheet on about 8 days a year.

1. Introduction

Since March 1955 it has been the practice at the R.A.E., Farnborough, on working days to fly a morning weather reconnaissance flight at about 0800 hr clocktime. The aircraft used were drawn from different flights but usually carried an observer from the Meteorological Research Flight. The aircraft did not usually carry meteorological instruments and the purpose of the flight was to make a broad survey of cloud conditions over Southern England. It was usual to make a rapid ascent over Farnborough at about 3,000 ft min^{-1} to above the highest cloud top followed by flight on a bearing towards which significantly different cloud conditions appeared, or were expected, to exist. Cloud conditions and any other items of meteorological interest were recorded by the observer and entered in a log book at the end of the flight. Over the years there has accumulated a body of data which is by no means ideal investigational material but which, nevertheless, contains some information of interest.

In this paper the occurrence of extensive layer cloud in various height bands, and also in the temperature band 0 to − 15°C, has been investigated. The latter band was chosen as likely to be of value in assessing how frequently supercooled layer cloud may exist, a matter of interest in aircraft icing problems and cloud modification studies. The choice of − 15°C was made from two considerations : (1) statistics of occurrence of *clear* ice formation on aircraft (Jones 1956) show a significant decrease in frequency at temperatures below − 15°C, implying that clouds composed *wholly* of supercooled water may be rare below that temperature and (2) earlier data on layer clouds (Mason 1957, p. 212) ' suggest that, on average, a sharp rise in the production of ice crystals occurs in the temperature range − 12°C to − 16°C,' so that sufficient ice crystals may exist in layer clouds extending to temperatures below − 15°C to make any modification by seeding difficult to detect.

2. Data used

Records from 1 March 1955 to 10 January 1958 inclusive were examined. In this period flights were made on 599 days representing 57 per cent of the total number of days. On a small number of occasions weather conditions were the reason for a flight not being made (e.g., fog, strong cross-wind or particularly fine and settled weather over a wide area) but the majority of days missed are accounted for by non-working days and it is thought that no bias is introduced by regarding the flights as typical of the whole period under study or any similarly long period.

Because the flights followed no set plan, attention was confined to that part of the flight which was virtually the same from day to day, namely the ascent from Farnborough. For each day the base and top of all layers (i.e., excluding convection cloud) of cloud reported as being 4/8 or more were recorded diagrammatically using pressure levels for ease of comparison with upper-air ascents. 4/8 or more cloud as reported from the aircraft was taken as indicating extensive cloud which might well become 8/8 in some direction from the location observed or appear to be 8/8 when viewed from the ground. (In the analysis of Section 5 of this paper, reports of 8/8 cloud are treated separately).

On the same diagram were plotted the pressures of the 0°C and — 15°C isotherms from the 0200 GMT Crawley ascent from 1 March 1955 to 31 March 1957 and the 1100 GMT Crawley ascent from 1 April 1957. No attempt was made to interpolate in time or distance since it was thought that the errors so introduced might well be as great as using the actual ascents and that interpolation was not therefore worth the labour involved.

From continuous diagrams of the type described, covering the whole period under review, it was easy to extract the cloud statistics of the following sections.

3. Frequencies and depths of 4/8 or more of layer cloud in various height ranges

(a) *Frequency of cloud*

Table 1 gives the percentage frequency with which 4/8 or more of layer cloud occurred somewhere in the pressure ranges indicated on the 599 flights. The same cloud layer can, of course, be recorded in more than one pressure range. Less than 4/8 layer cloud at any level was reported on 26·4 per cent of flights; on many of these occasions convection cloud was present or likely to develop later.

TABLE 1. Layer cloud frequencies in various pressure ranges

Pressure range (mb)	1,000 — 900	900 — 800	800 — 700	700 — 600	600 — 500	500 — 400	400 — 300	300 — 200	200 — 170
Percentage frequency	44·2	38·4	27·6	20·7	19·1	20·7	19·5	15·0	1·0
Pressure range equivalent in km	0·12 — 0·97	0·97 — 1·95	1·95 — 3·02	3·02 — 4·17	4·17 — 5·55	5·55 — 7·16	7·16 — 9·17	9·17 — 11·80	11·80 — 12·80
Percentage frequency per km height	52·0	39·2	25·8	18·0	13·8	12·8	9·7	5·7	1·0

Fig. 1 displays the same information in the form of percentage frequencies per unit height range. The chance of encountering cloud in any km layer of the atmosphere decreases from about 1 in 2 flights in the lowest kilometre to 1 in 5 flights around 3 km height and 1 in 13 flights at a height of 9 km. Above this level the information may be unreliable since the aircraft did not always ascend higher but Murgatroyd and Goldsmith (1956) have shown that high cloud is likely above a station in Southern England on 47 per cent of occasions and on 32 per cent of these the base is likely to be above the 300 mb level; i.e. on about 15 per cent of flights one would expect cloud above the 300 mb level, in remarkable agreement with Table 1. We may therefore regard Fig. 1 as reasonably reliable up to about 12 km when the chance of encountering cloud in a 1 km layer appears to be about 1 in 50.

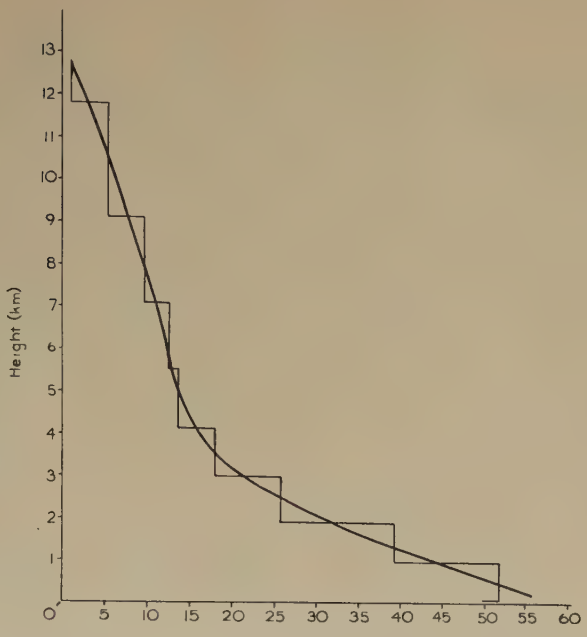

Figure 1. Layer cloud frequencies – variation with height.

(*b*) *Frequencies of cloud thicknesses*

Table 2 gives the percentage frequencies of occurrence of different cloud depths for cloud layers of which the bases are in various pressure intervals. More than one layer sometimes occurred with bases in the same pressure interval.

TABLE 2. PERCENTAGE FREQUENCIES OF CLOUD THICKNESSES

		Cloud thickness (mb)												
Cloud base (mb)	Total no. of layers	0 — 50	50 — 100	100 — 150	150 — 200	200 — 250	250 — 300	300 — 350	350 — 400	400 — 450	450 — 500	500 — 550	550 — 600	>600
1,000-900	279	58·1	21·5	6·1	3·6	2·5	3·2	1·1	0·7	0·4	0	0·7	0·7	2·2
900-800	126	64·3	19·8	3·2	4·0	0·8	0	1·6	1·6	3·2	0	0·8	0	0·8
800-700	93	50·5	22·6	3·2	5·4	2·1	2·1	2·1	3·2	3·2	4·3	1·1	0	0
700-600	45	57·8	13·3	4·5	4·5	11·1	0	2·2	4·5	0	2·2	0	0	0
600-500	57	49·1	22·8	10·5	5·3	5·3	5·3	1·8	0	0	0	0	X	X
500-400	57	35·1	22·8	14·0	12·3	14·0	1·8	0	0	0	X	X	X	X
400-300	46	43·5	30·5	23·9	2·2	0	0	0	X	X	X	X	X	X
300-200	24	70·8	29·2	0	0	0	X	X	X	X	X	X	X	X

Fig. 2 displays the same information, with the pressure thicknesses converted to km, in the form of cumulative percentage frequencies of layers exceeding various thicknesses. The greater proportion of layers exceeding one or two km depth when the cloud base is high, than when it is low, is striking. Thus about half the layers encountered with bases between 500 and 300 mb (5·5 to 9·2 km) were more than 1½ km deep but only one-quarter of those with bases between 800 and 500 mb (2 to 5·5 km) and one-seventh of those with bases up to 2 km high. The greater frequency of layers of all kinds at the lower heights, however, results in the frequency of encountering a layer more than 1½ km deep being about 7 to 8 per cent at all heights.

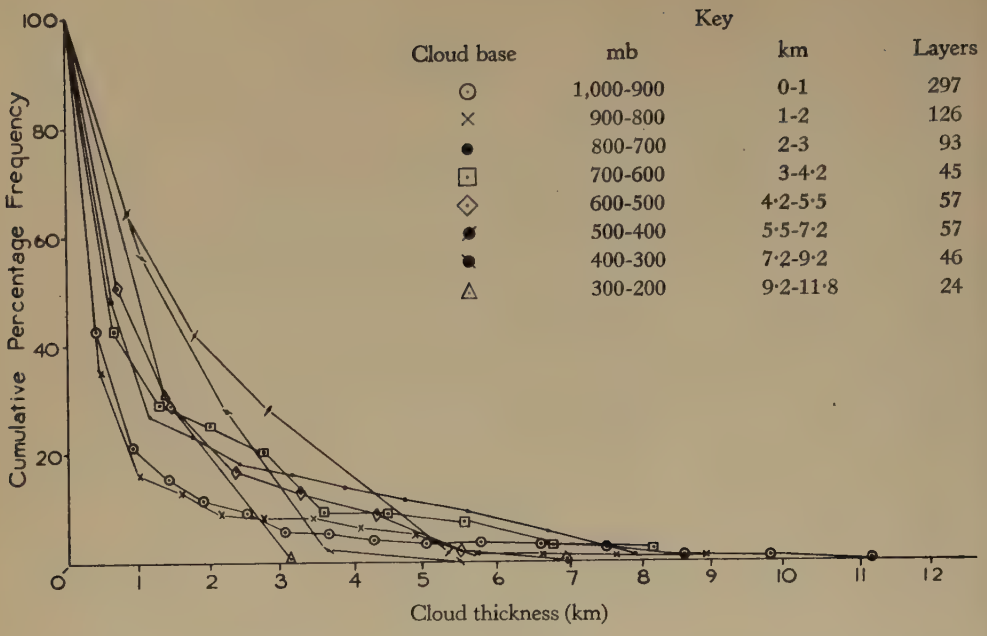

Figure 2. Percentage frequencies of layer cloud thickness for different heights of cloud base.

(c) *Cloud thicknesses above the melting level*

The depth of cloud likely to be encountered above the melting level and its frequency of occurrence is of interest to aircraft designers and others. Table 3 gives these frequencies for the 599 flights.

TABLE 3. FREQUENCIES OF VARIOUS CLOUD THICKNESSES (mb) ABOVE THE MELTING LEVEL

Cloud base or height of 0°C isotherm (whichever is higher) (mb)	Total no. of layers	Cloud thickness above melting level (mb)												
		0 — 50	50 — 100	100 — 150	150 — 200	200 — 250	250 — 300	300 — 350	350 — 400	400 — 450	450 — 500	500 — 550	550 — 600	> 600
1,000-900	17	8	6	1	1	0	1	0	0	0	0	0	0	0
900-800	64	32	13	2	3	3	2	0	2	2	0	2	2	1
800-700	86	36	18	3	5	6	1	1	8	3	4	1	0	0
700-600	52	31	6	3	3	4	0	1	2	2	0	0	0	0
600-500	57	28	13	6	3	3	3	1	0	0	0	0	0	X
500-400	57	20	13	8	7	8	1	0	0	0	0	X	X	X
400-300	46	20	14	11	1	0	0	0	0	X	X	X	X	X
300-200	24	17	7	0	0	0	0	X	X	X	X	X	X	X

It is of interest to note that although 279 layers of cloud with base between 1,000 and 900 mb were encountered on the flights (Table 2) only 17 of these (6·1 per cent) were associated with a melting level below 900 mb although 14·5 per cent of all flights were made with the melting level so low. Only three of the 17 layers exceeded 100 mb (1 km) in thickness. Thus layer cloud with base up to 1 km is less frequent when the melting level is below 1 km and when it does occur is rarely more than 1 km thick.

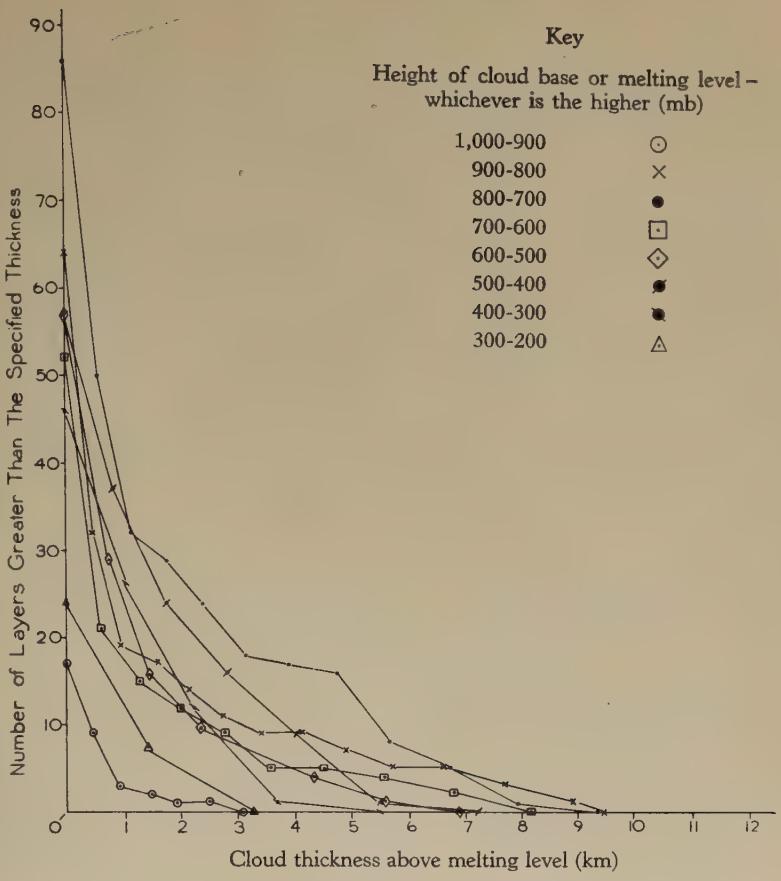

Figure 3. Frequencies of cloud thicknesses above melting level.

Fig 3 portrays the same information in the form of the number of layers encountered greater than specified thicknesses in km. Summing individual layer totals from these curves, which are based on 599 flights, we can determine the number of times in 100 flights when there is likely to be cloud above the melting level exceeding a specified thickness. This is shown in Table 4. For small thicknesses more than one layer may exist on one flight which accounts for the high frequency given.

TABLE 4. FREQUENCY OF ENCOUNTERING VARIOUS THICKNESSES OF LAYER CLOUD ABOVE THE MELTING LEVEL ON 100 ASCENTS

Cloud thickness above melting level (km)	0	1	2	3	5	7	9	10
No. of times per 100 flights thickness exceeded	67·2	28·4	18·1	11·1	5·1	1·7	0·15	0

4. LAYER CLOUD WITH TEMPERATURES BETWEEN 0°C AND − 15°C

The frequency of occurrence of 4/8 or more layer cloud at temperatures between 0 and − 15°C on the 599 flights is shown in Table 5.

TABLE 5. No. OF FLIGHTS WHEN LAYER CLOUD EXISTED IN THE TEMPERATURE BAND 0 TO − 15°C

Wholly within the band	Base temp > 0°C Top temp > − 15°C	Base temp > 0°C Top temp < − 15°C	Base within band Top temp < − 15°C	Total
92	43	23	42	200

Although there was layer cloud to be found somewhere between 0 and − 15°C on 33 per cent of days this does not mean that there was always substantial supercooled water in the clouds with this frequency. When the cloud top temperature was below − 15°C (on 65 occasions) it was very likely that the cloud was infected with ice crystals and on some of the other occasions the cloud layer may well have been seeded by ice crystals from a layer above.

The greatest chance of substantial supercooled water exists when the cloud-top temperature is − 15°C or higher and there is no other cloud layer above. This was found on 79 flights, 13·2 per cent of all flights. Because these occasions are of special interest they have been examined in greater detail in Section 5.

5. LAYER CLOUD WITH TEMPERATURE AT TOP BETWEEN 0°C AND − 15°C WITH NO CLOUD ABOVE AT TEMPERATURES BELOW − 15°C

(*a*) *Temperature distribution*

Because on some days more than one layer qualified there was a total of 94 layers on the 79 days distributed as in Table 6.

TABLE 6. CLOUD-TOP TEMPERATURE

	− 0·5 to − 5°C	− 5·5 to − 10°C	− 10·5 to − 15°C	Total
No. of layers 4/8 or more	41	25	28	94
8/8 only	16	12	14	42

(*b*) *Water contents*

As an indication of the relative maximum water contents of these clouds we can determine the amount of water set free by adiabatic ascent from cloud base to cloud top. This will probably be a considerable overestimate of supercooled water content but being related directly to temperature and pressure at cloud base, and total cloud thickness is a useful parameter for comparison between clouds. The frequencies of occurrence of various water contents are shown in Table 7.

TABLE 7. FREQUENCY OF OCCURRENCE OF ADIABATIC WATER CONTENTS

Cloud amount	Adiabatic water content (g m^{-3})							
	< 0·1	0·1 — 0·5*	0·6 — 1·0	1·1 — 1·5	1·6 — 2·0	2·1 — 2·5	2·6 — 3·0	> 3·0	Total
4/8 or more	8	38	22	9	6	5	2	4	94
8/8 only	4	11	10	2	5	4	2	4	42
8/8 only with top temperature below − 5°C	1	8	9	2	2	4	0	0	26

(c) *Discussion of Tables 6 and 7*

Table 6 shows that of the 94 occasions when supercooled layer cloud may be presumed to have existed only 42 were so extensive as to be reported as unbroken when seen from the air. Lewis (1951) reports that in the United States of America 66 per cent of supercooled water contents of stratus and stratocumulus clouds were less than 0·3 g m^{-3} with a mean value of about 0·25 g m^{-3} and similar values are reported from Russia (Minervin, Mazin and Burkovskaya 1958) when mean values at temperatures between 0 and — 15°C ranged from 0·11 to 0·34 g m^{-3} with maximum values as high as 1·53 g m^{-3}. These suggest that the adiabatic water contents of Table 7 overestimate the actual *mean* values by a factor of 3 to 4 (Lewis also reports observed values averaging between one-fourth and one-half of the calculated values). A supercooled water content of 0·5 g m^{-3} would give a rate of ice accretion on the leading edge of an aircraft travelling at a speed of about 270 kt of about 0·4 cm min^{-1} (Jones 1956) and we see from Table 7 that this rate of ice accretion might have been exceeded on 15 of the 42 occasions of unbroken cloud, but on only 6 of them at temperatures below — 5°C. Since a speed of 270 kt might be expected to give a protection by kinetic heating from icing at temperatures down to about — 5°C, we see that severe clear ice on such an aircraft (regarding 0·4 cm min^{-1} as a severe icing rate) might be encountered on about one per cent of all flights. Supercooled water contents as high as 1 g m^{-3} would be expected with a frequency of about 0·7 per cent at temperatures between 0 and — 5°C but much less frequently at lower temperatures.

It should be borne in mind, however, that severe ice formation can also occur in mixed clouds containing supercooled water and ice crystals.

The temperature below which silver iodide begins to be effective as a seeding agent is about — 5°C, but it is about 1,000 times more effective at — 10°C (Mason 1957, p. 159). Table 6 shows that on the 599 flights there were 26 layers of unbroken cloud in which silver iodide might reasonably be expected to have some effect although the effect might only be readily detectable (e.g., by thinning or dissipation of cloud and/or precipitation from the cloud) in 14 layers. This latter figure represents a probability of 0·023 or 8·4 days a year.

Acknowledgments

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The fallspeeds of hailstones

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SUMMARY

Measurements have been made of the speeds of ice objects resembling hailstones during free fall, and of the drag coefficients of such objects and of natural hailstones. It has been found that the drag coefficients range from those of spheres (about 0·45) to about 0·8, depending on the shape and surface characteristics of the stone.

1. INTRODUCTION

A knowledge of the fallspeeds of hailstones is important because they determine the growth rates of stones and the size of the largest which can be grown in a given updraught. Published computations of the growth of hailstones have been based on the drag coefficients of spheres (e.g., Bilham and Relf 1937, Schumann 1938 and Ludlam 1958) but, in reviewing the aerodynamic properties of hailstones, Weickmann (1953) suggests that regular stones have a drag coefficient of about one and that particles of irregular shape may have drag coefficients as much as three times greater. Over the range of Reynolds numbers appropriate to hailstones of diameter more than 1 cm, the drag coefficient for spheres is approximately constant, having a value of $0{\cdot}45 \pm 0{\cdot}03$ (Goldstein 1938); at a Reynolds number of about 2×10^5, transition to turbulence occurs in the boundary layer and the drag is greatly reduced. This is thought to impose an upper limit of about 10 cm to the diameter of hailstones (Bilham and Relf 1937). However, if Weickmann's suggested values are correct, then the fallspeeds of hailstones may be less than half those of spheres of the same mass.

List (1959) has recently reported experiments in which he determined the drag coefficients of four hailstones having masses between 18 and 32 g. For stones which were roughly spherical he obtained drag coefficients of 0·6, while for oblate stones (axis ratios 1 : 0·8 : 0·6) he obtained values of about 0·7. These are considerably lower than the values proposed by Weickmann, but higher than the value appropriate to spheres.

This paper describes experiments carried out to determine the fallspeeds of model hailstones during free fall, using a GL7 lock-follow radar, and the drag coefficients of both model and natural stones under controlled conditions in a wind tunnel.

2. MEASUREMENTS MADE DURING FREE FALL

2.1 Procedure

Model stones were made by freezing water in hemispherical cups and then cementing the two halves together. Within each hemisphere were frozen a number of strips of aluminium foil (radar ' window ') to increase the back-scatter cross-section. A stone was placed in a cardboard (non-reflecting) box suspended beneath a 100 g meteorological balloon inflated with hydrogen; a second 10 g balloon, partially inflated with air, was attached to the bottom of the box (Fig. 1). A fuse, made by soaking loose string in a concentrated solution of sodium nitrate, was tied to the suspending cord and its length adjusted so that at a height of about 2 km the box containing the ice particle was separated from the lifting balloon. The lower balloon then served as a parachute, tipping the box upside down and ejecting the stone.

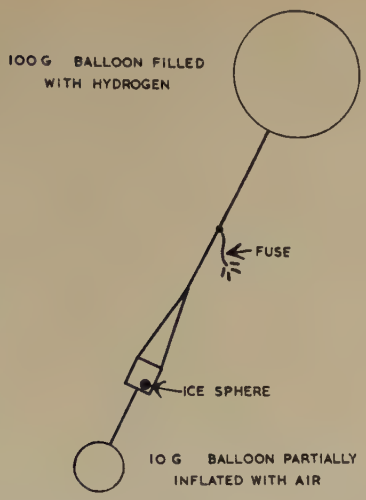

Figure 1. Apparatus used to drop the model hailstones in the free-fall experiments.

Once the radar had found the target, there was no difficulty in following both the ascent and descent of the stone, even though the latter was considerably faster. The radar was instrumented for wind finding, and the range, azimuth and elevation were recorded automatically every five seconds.

Of the 17 stones with which measurements were successfully made, three were virtually spherical, five had axes in the ratio 1 : 1 : 0·7-0·9, and the remaining three in the ratio 1 : 1 : 0·43-0·52. The surfaces of the stones were relatively smooth with no major projections, but in some instances were made with surface depressions, and similar irregularities, comparable with those found on natural hailstones (see Ludlam and Macklin 1959). The stones were chilled to prevent melting as far as possible, but it is probable that a superficial melting occurred during their fall. The mass, dimensions, and the condition of the surface of each stone were recorded prior to each flight.

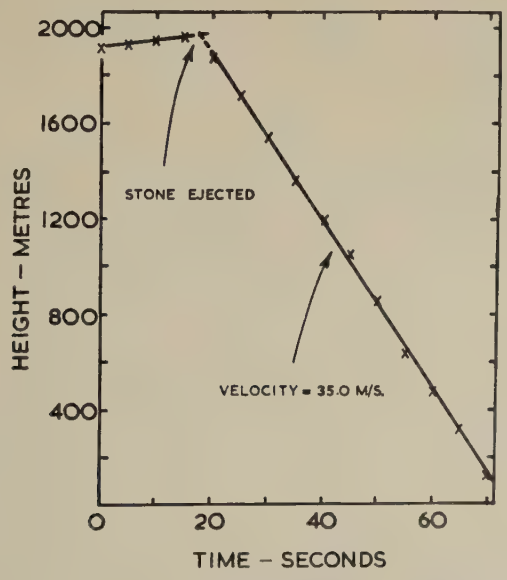

Figure 2. Typical example of the height-time trajectory of an ice sphere.

2.2 Results

From the radar records a plot of the height of each stone against time was made, as shown in Fig. 2. The accuracy with which the fallspeeds were determined from such diagrams was ± 1 m sec^{-1}, except in a few instances where the fall of the stone was observed for only a short period or where there was a discontinuity in the height-time relationship, presumably due to a re-orientation of the stone.

Fig. 3 shows the measured fallspeed as a function of the mass of the stone; where the error is greater than ± 1 m sec^{-1} the range of values is indicated. Superimposed on this diagram is a curve based on the drag coefficients for spheres of density 0·9 g cm^{-3}, the density of the ice particles used. Most of the values are within ± 20 per cent of the curve for spheres and there was no consistent effect produced by the surface irregularities or by small eccentricities. The scatter in the velocities is considered to be due, at least in part, to slightly different orientations of the objects as they fell. The smaller oblate stones, however, had fallspeeds considerably less than spheres of the mass same. The drag coefficient for these stones, calculated from their dimensions, mass and fallspeed and the air density (deduced from the appropriate aerological sounding), was found to be about 0·8. The accuracy of this method is not great (about ± 25 per cent), as it was not possible to determine the orientation of the particle during its fall. However, the

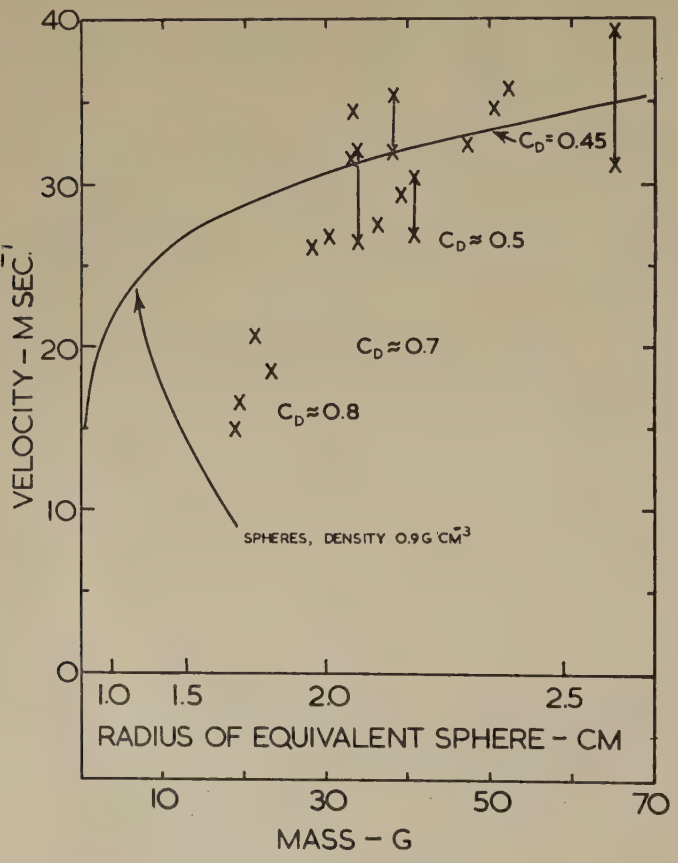

Figure 3. The free-fall speed of a stone as a function of its mass. A scale showing the radius of the sphere of equal mass is also given. The curve for spheres is based on a mean drag coefficient of 0·45 and an air density of $1{\cdot}15 \times 10^{-3}$ g cm^{-3}, the latter value being appropriate to the present experimental conditions. Drag coefficients, estimated from the data, are indicated.

alue obtained suggests that, while the drag coefficients of smooth spherical stones approxiiate to those accepted for spheres, more oblate shapes have drag coefficients which may e as much as twice those of spheres of the same mass. Accordingly, it was decided to arry out a series of measurements of the drag coefficients of both model and natural ailstones, under controlled conditions.

3. Measurements of drag coefficients

3.1 Procedure

The objects to be tested were mounted on a right-angled (' sting ') support in a vind tunnel which could produce airspeeds between 12 and 40 m sec^{-1}. The tunnel vas equipped for making drag measurements with a balance (which could be read to ± 0·5 g) and an anemometer which measured the pressure head from a pitot tube inserted n the airstream (to ± 0·05 mm of water). The room temperature and pressure were ecorded and airspeeds could be estimated to ± 5 per cent although, for the purpose f determining the drag coefficients, the direct measurement of the pressure head was ufficient (see below). Measurements were made at intervals in wind speed of 5 m sec^{-1} ver a range of 12 to 37 m sec^{-1}.

In order to check the balance calibration and to take into account the effect of the irstream on the exposed portions of the support, tests were carried out using accurately onstructed smooth spheres ranging in size from 3·5 to 7·5 cm in diameter. An independent alibration of the balance alone was made by exerting a known horizontal force on the upport.

The models tested were constructed either of plasticine moulded over a wooden ormer to give rigidity, or ice formed by freezing water in hollow rubber spheres. In oth cases variations in shape were readily produced. A short piece of rod was inserted n each object and this was used to locate it on the support. The object was rotated relative o the airstream by changing the position of this rod and three orientations were employed, differing by 45°. The cross-sectional areas presented to the airstream were obtained hotographically.

The shapes of the models and nature of the surface irregularities were deliberately varied to ascertain their respective effects on the drag coefficient. The plasticine models were approximately ellipsoidal with the following ratios of axes : 1 : 1 : 1, 1 : 1 : 0·8, 1 : 1 : 0·67, 1 : 1 : 0·46, 1 : 0·84 : 0·75 and 1 : 0·75 : 0·68; the maximum dimension f each was between 5 and 7·5 cm. The models were given smooth surfaces, surface epressions, and projections. The models made of ice had a variety of shapes and sizes n approximately the same range of eccentricities as those of plasticine, but their diameters were smaller, ranging from 3·5 to 6 cm (masses 27 to 185 g); tests were not made on ce spheres with projections. The ice models (and the hailstones tested) were coated with a thin plastic film (formed by dipping them into a one per cent solution of formvar in thylene dichloride) and chilled in dry ice prior to each observation, to prevent excessive melting. Since large hailstones tend to grow with wet surfaces (Ludlam 1958) observations were made to see whether the drag coefficient was affected if the surface of the models was wet.

Measurements were also made on three moderately large hailstones having masses of 14·7, 19·6 and 21·8 g, and ratio of axes 1 : 0·89 : 0·70, 1 : 1 : 1, and 1 : 0·77 : 0·45 respectively. It is believed that non-spherical stones fall with their shortest axis vertical. When hail stones grow with a wet surface the excess water is forced round the sides to the back of the stone so that it grows as an ellipsoid; the front accreting surface is consequently smooth, whilst the back is irregular and knobbly (Ludlam and Macklin 1959). List (1959) has carried out experiments in a water tank which confirm this direction of fall. The measurements of the drag coefficients of the oblate hailstones made in the present experiments were confined to air flow in this direction.

3.2 *Results*

The drag coefficient, C_D, is given by the following relation

$$D = \tfrac{1}{2}\rho_a V^2 A C_D \quad . \quad . \quad . \quad . \quad (1)$$

where D is the drag force, ρ_a is the density of air, V is the air speed and A is the cross-sectional area normal to the airstream. The quantity $\frac{1}{2}\rho_a V^2$ was obtained directly from the manometer measurements since

$$\tfrac{1}{2}\rho_a V^2 = \rho_w g h \quad . \quad . \quad . \quad . \quad (2)$$

where ρ_w is the density of water, g is the acceleration due to gravity and h is the pressure head.

3.2.1 *Tests on smooth spheres*

The measurements of the drag coefficients of spheres were made primarily to calibrate the equipment over the range of conditions of the experiments. The systematic error introduced by the effect of the airstream on the exposed parts of the upright support to the balance was removed by assuming that the drag coefficients for spheres are those given by Goldstein (1938, p. 16). The expected drag force appropriate to the conditions of a measurement was deduced from these values, the actual drag force measured and the difference attributed to the effect of the airstream on the support. A mean correction based on the results for all three spheres was then deduced; this amounted to a 10 to 20 per cent reduction in the measured values of the drag force, depending on the air speed. The drag coefficients of the spheres were made for three different diameters over partly coincident ranges of the Reynolds number with a consistency which can be seen in Fig. 4; it is considered that the method employed gives values which are generally accurate to better than $\pm$ 10 per cent.

3.2.2 *Tests on plasticine models*

The values for the drag coefficients for the plasticine models are summarized in Table 1. The nature of the surface is described as ' smooth ' when no deliberate attempt was made to introduce surface irregularities, as ' with depressions ' when roughly circular depressions about 2 cm across and 0·75 cm deep, covered the surface, and ' with projections ' when

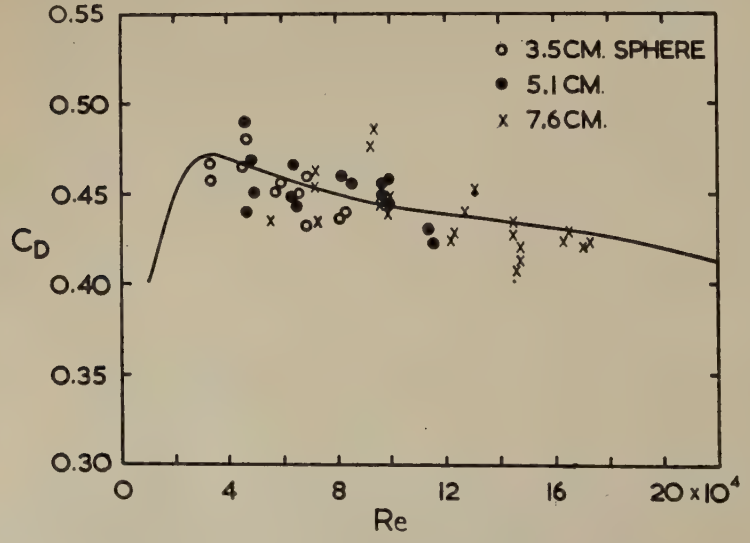

Figure 4. The drag coefficients obtained for smooth spheres. The curve shown is that given by Goldstein (1938).

lunt pointed projections, having a base about 1 cm across, were introduced randomly ver the surface about 3 to 4 cm apart. The orientation of the ellipsoidal objects is desig- ated 0° when the shortest axis was in the direction of the airstream, and 90° when the ongest axis was in this direction. With these and the irregular objects the dimension sed to determine the Reynolds number was the diameter of a circle whose area was qual to that presented by the object to the airstream.

TABLE 1. RESULTS FOR THE PLASTICINE MODELS

Ratio of axes	Nature of surface	Orientation	Approx. range of 10^{-4} Re*	C_D
1 : 1 : 1 (sphere)	(1) smooth		7-17	0·45-0·21
	with depressions			0·50-0·21
	(2) smooth		5·5-13	0·53-0·25
	with projections,			
	0·5-1·0 cm			0·58-0·41
	1·5-2·0 cm			0·64-0·52
1 : 1 : 0·8	smooth	0°	7-17	0·55-0·40
		45		0·51-0·44
		90		0·48-0·17
	with depressions	0	7-17	0·54-0·40
1 : 1 : 0·67	smooth	0	7-15	0·61-0·57
		45		0·60-0·53
		90		0·36-0·19
	with depressions	0	7-15	0·80-0·70
1 : 1 : 0·46	smooth	0	7-18	0·67-0·61
		45		0·61-0·56
		90		0·33-0·15
	with depressions with projections,	0		0·64-0·50
	0·5-1·5 cm	0		0·70-0·61
1 : 0·84 : 0·77	smooth	0	7-16	0·59-0·35
		45		0·52-0·41
		90		0·31-0·13
	with depressions	0		0·59-0·41
1 : 0·75 : 0·68	smooth	0	6-14	0·48-0·35
		45		0·50-0·25
		90		0·39-0·16
	with depressions	0		0·70-0·35

the dimension used in determining the Reynolds number of an irregular object was the diameter of the circle of the same area of cross-section.

The values of the drag coefficient of the plasticine spheres descreased from about ·5 to 0·2 as the Reynolds number increased from 7 to 17 × 10^4. In the critical range of Reynolds numbers in which transition to turbulence occurs in the boundary layer, the nature of the surface plays an important part in determining the drag coefficient. The oughness parameter (the ratio of the size of the surface irregularities to the diameter) or the plasticine sphere was about 0·02, and the drag coefficients obtained here are con- istent with those determined by other workers (see Hoerner 1958, p. 3-8). A similar ffect may occur with the ellipsoidal models, and this must be considered when applying his data to objects with surfaces of different character.

It has been indicated that hailstones grow with their shortest axis vertical, so that the most relevant data in Table 1 are those for an orientation of 0°. As the objects were rotated from 0 to 90° the drag coefficients decreased; this is to be expected as the objects then presented a more streamline shape to the airflow. As the objects became more oblate the drag coefficients increased, the values increasing from 0·45 to 0·67 as the shortest axis decreased to a value about one-half that of the other two axes. Shortening a second axis (objects 5 and 6) also reduced the drag coefficients, particularly at high Reynolds numbers.

The effect of surface depressions and protuberances was not consistent throughout the series of measurements. In the case of the third and sixth objects there was a significant increase in the drag coefficient when the surface depressions were introduced, while in the case of the fourth object there was a reduction in the drag coefficient (although in view of the experimental error it is not possible to tell just how significant this decrease was). The drag coefficient of the sphere was increased markedly by the protuberances, the increase being dependent on their size, while with the fourth model, the relatively high drag coefficient was virtually unaffected.

3.2.3 *Tests on ice models*

The values of the drag coefficients for the ice models are given in Table 2; measurements were made only when the shortest axis was in the direction of the airstream. The results show a similar trend to those of the plasticine models, the values increasing as the objects become more oblate. The values for the spheres are comparable with those of plasticine, but do not decrease as rapidly with increasing Reynolds number. This is attributed to differences in surface texture. The results for ellipsoids having similar axis ratios in the two series of measurements are also comparable.

TABLE 2. RESULTS FOR ICE MODELS

Ratio of axes	Nature of surface	Approx. range of 10^{-4} Re	C_D
1 : 1 : 1	smooth	7-12	0·54-0·41
	smooth	6-14	0·53-0·46
	minor irregularities	6-14	0·47-0·50
	smooth	5-11	0·62-0·51
	smooth	7	0·50
1 : 1 : 0·8	smooth	7	0·54
1 : 0·96 : 0·87	with depressions	7	0·54
1 : 1 : 0·75	smooth	5-11	0·53-0·54
1 : 1 : 0·88	smooth	6-10	0·56-0·53
1 : 1 : 0·81	minor irregularities	8-16	0·50-0·48
1 : 1 : 0·76	smooth : asymmetrical	5-13	0·53-0·50
1 : 1 : 0·68	asymmetrical with depressions	7-12	0·63-0·57
1 : 0·91 : 0·64	smooth	6-14	0·65-0·54
1 : 0·86 : 0·60	with depressions	4-8	0·61
1 : 0·80 : 0·57	with depressions	5-13	0·62-0·55
1 : 0·85 : 0·62	with depressions	5-9	0·68-0·66

Several observations were made with various ice models to ascertain whether the state of the surface (dry or wet) affected the drag coefficient. No detectable change was observed on any occasion when the surface was allowed to become wet during the course of an experiment. It is interesting to note that the diameters of the droplets shed in the wake were about 2 to 3 mm at a speed of 15 m sec^{-1}, decreasing to less than 1 mm at 30 m sec^{-1}.

3.2.4 *Measurements made with hailstones*

Details of the determinations of the drag coefficients of the three hailstones are hown in Table 3 together with those made by List (1959). In the present experiments he probable orientation of the stone during free fall was decided by inspection; List ives the drag coefficients for both orientations with the shortest axis in the direction f the airstream. The agreement between the values for the hailstones, those for models f the same shape, and List's values is good, and supports the view that it is primarily he shape which determines the drag coefficient of large hailstones.

TABLE 3. RESULTS FOR HAILSTONES

Mass (g)	Axes (cm) and ratio of axes	velocity (m sec^{-1})	10^{-4} Re	C_D	Fallspeed (m sec^{-1})
					at 15°C, 1,000 mb
21·8	5·2, 4·0, 2·35	11·4	3·4	0·73	
	(1 : 0·77 : 0·45)	15·2	4·5	0·75	20 ± 1
		20·5	6·0	0·68	
		25·3	7·5	0·71	
19·6	3·6	11·3	2·6	0·61	
	spherical	14·7	3·9	0·63	25
		20·3	4·7	0·53	
		25·4	5·9	0·51	
	(second	14·8	3·9	0·59	
	orientation)	19·8	4·6	0·53	
		25·1	5·9	0·55	
14·7	3·7, 3·3, 2·6	11·4	2·6	0·74	
	(1 : 0·89 : 0·70)	14·5	3·3	0·66	21
		20·3	4·6	0·59	
		25·4	5·7	0·66	
					at the Weissfluhjoch Observatory (2,665 m above sea-level)
IST'S VALUES					
24·3	1 : 0·79 : 0·44			0·73	
				0·65	20
32·4	1 : 0·78 : 0·55			0·65	23
				0·70	
18·6	approximately			0·70	23
	spherical			0·57	
				0·62	
				0·64	24·5
				0·54	
				0·53	27
23·3	1 : 0·77 : 0·64			0·62	
				0·62	23

The fallspeeds given in the last column of Table 3 were obtained by plotting the easured value of the drag force against the corresponding air speed. The speed appro- riate to a drag force equal to the weight of the stone was then determined; the self- onsistency of the plotted points suggests an accuracy of ± 1 m sec^{-1}. The fallspeeds uoted for List's experiments are taken from this paper and are appropriate to the con- itions at the Weissfluhjoch Observatory (2,665 m above sea level).

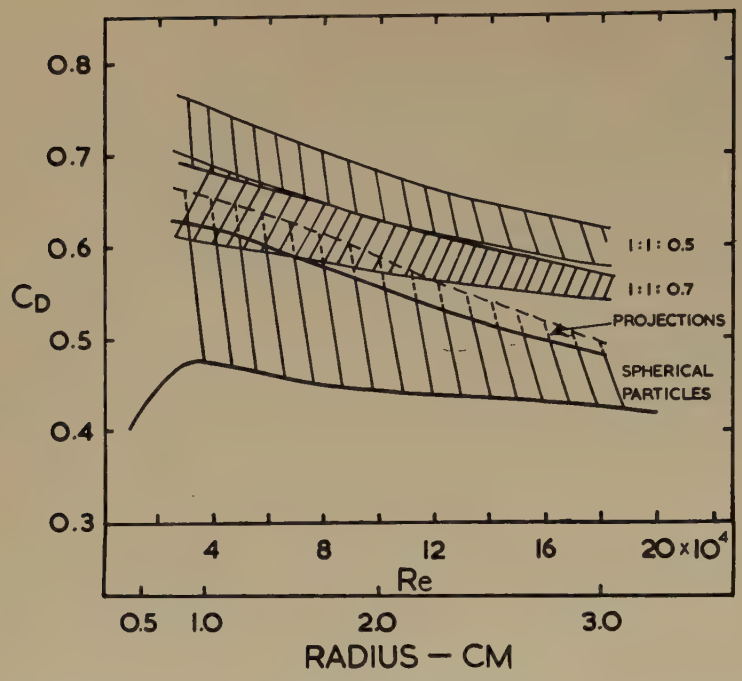

Figure 5. Proposed scheme for determining the drag coefficients of hailstones, with an approximate scale of hailstone radius obtained by assuming $C_D = 0{\cdot}6$, $T = -10°C$ and $p = 600$ mb.

4. Summary and conclusions

The drag coefficients of hailstones range from about 0·45 (for smooth spheres) to about 0·8, depending mainly on the shape and the Reynolds number, and partly on the nature of the surface of the stone. The values obtained for spherical stones were in general greater than those for smooth spheres, although in some instances comparable values were obtained. Fig. 5 depicts a scheme, based on the above measurements, for determining the drag coefficients of hailstones. It is only approximate, however, and individual particles may have drag coefficients differing appreciably from those indicated; for example, the values obtained for the third plasticine model (1 : 1 : 0·67) when surface irregularities were introduced are higher than those shown in the diagram. The present data indicate that the drag coefficients for hailstones, both of regular and irregular form, are considerably lower than those envisaged by Weickmann (1953). Since large hailstones are often asymmetrical, a reasonable mean value for the drag coefficient of hailstones with diameters greater than 1 cm is 0·6, which implies fallspeeds 15 to 20 per cent less than those of spheres of the same mass.

No attempt was made during these experiments to determine the conditions under which transition to turbulence occurs in the boundary layer for stones of various shapes (this may determine their maximum possible size) as the Reynolds numbers required were difficult to attain. The surface characteristics of the stone must be expected to play an important part in this effect.

Acknowledgments

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The expansion of clusters of particles in the atmosphere

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SUMMARY

In this paper, the problem of the spatial expansion of a cluster of particles under the action of turbulence is investigated. The basic difficulty, that this form of diffusion depends on the Lagrangian properties of the turbulent field, is met by the assumption that the Lagrangian and Eulerian time-correlograms are functionally similar. In consequence, it is possible to solve the equation governing the cluster's expansion to give prediction laws expressed only in terms of easily measured parameters of the turbulence. The application of the full solution is lengthy but a simple working approximation follows from the fact that over an important part of the expansion at which the size of the cluster is of the same order of magnitude as the Eulerian length-scale, the rate of expansion is approximately constant and is proportional to the square of the intensity of turbulence whilst independent of the length scale itself. Experiments to test these formulae have been carried out on the diffusion of clusters of Lycopodium spores over a few hundred metres downwind travel near the ground, and the diffusion of long crosswind clouds of aircraft-released fluorescent particles, sampled along the vertical, after several kilometres travel in the free atmosphere. Whilst the experiments are few in number and the conditions often did not strictly conform to the conditions of isotropy and homogeneity required by the theory, the results are satisfactorily consistent with the prediction formulae and imply an effectively constant ratio between the scales of the Lagrangian and Eulerian time-correlograms.

NOTATION

b – ratio of the Lagrangian length-scale and Eulerian length-scale (as defined by Mickelsen)

$E(k)$ – energy spectrum function (Eulerian)*

$E_1(k)$ – spectrum corresponding to covariance $R_1(s)$

$E_2(k)$ – spectrum corresponding to covariance $R_2(s)$

$F(k)$ – longitudinal one-dimensional spectrum function (Eq. 22)*

$G(k)$ – lateral one-dimensional spectrum function (Eq. 23)*

i – (other than as suffix) the full intensity of turbulence $= \sqrt{\overline{u^2}_\infty}/U$ (or when specifically referring to the cross-wind component v, $i = \sqrt{\overline{v^2}}/U$)

l – length-scale of a covariance function (unless otherwise stated it refers to the trace-covariance)*

n – frequency in cycles per unit time

r – magnitude of the vector separation $\mathbf{r}$ of two particles

$R_{ij}(r)$ – mean product of $u_i(\mathbf{x})$ and $u_j(\mathbf{x}+\mathbf{r})$ over all $\mathbf{x}$ ($\mathbf{x}$ and $\mathbf{r}$ two space vectors)*

$R_{ii}(r)$ – $R_{11}(r) + R_{22}(r) + R_{33}(r)$ following the usual repeated suffix notation*

$R(r)$ – $\frac{1}{3} R_{ii}(r)$ is the trace-covariance

R'_L – Lagrangian covariance of velocities u'_i

$R_1(s)$ – Eulerian covariance of velocities u'_i appropriate to equally positioned particles in two identical clusters separated by a distance s

$R_2(s)$ – Eulerian covariance of the mean velocities of these two clusters

t – time, with origin at moment of release of cluster

U – wind speed

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u_i – velocity component in the i^{th} direction

u_{ai} – velocity component in the i^{th} direction of the a^{th} particle

u'_i – i^{th} direction component of the velocity of a particle relative to the mean for the cluster

$< u_i >$ – mean velocity component of all the particles in the cluster

$\overline{u'_\sigma{}^2}$ – mean square value of u'_i for all the particles in the cluster

V – denotes the distribution of particles within the cluster

$V(\sigma k)$ – the spectral window, defined in Eq. (25), defining the rate of growth of the cluster

v – cross-wind velocity fluctuation

v_g – terminal velocity of particle

x – distance downwind from point of release of cluster

y – separation of two cross-wind points

β – the ratio of the Lagrangian time-scale and the Eulerian time-scale. Unless otherwise stated, $\beta = \beta_3$, the 3 denoting that the Lagrangian covariance is being compared with the Eulerian trace-covariance

θ – angular deviation of wind vane

k – wave number (following Batchelor (1953), k is the number of waves in 2π units of length)

ξ_i – i^{th} component of the vector separation of two particles

σ – standard deviation of the particle positions about the centre of the cluster

τ_σ – Lagrangian time-scale of the cluster, standard deviation σ

$$u_\sigma{}^{12}\,\tau_\sigma = \int_0^\infty R'_L(t)\,dt$$

(* a fuller definition is given by Batchelor 1953)

1. Introduction

In 1926, L. F. Richardson put forward new ideas which, in principle, have formed the basis of the attack on the problem of how rapidly an airborne cluster of particles expands spatially under the action of turbulence. This problem is in contrast with that of the turbulent spread of particles released serially from a fixed point. His analysis showed that the statistical rate of change of the separation of a pair of particles would, in general, be a function of their separation. This idea was later elaborated and developed by Batchelor who clearly distinguished between these two complementary aspects of diffusion. His 1952 paper in which the fundamental nature of the cluster problem is clarified, represents the diffusion of pairs of particles, or of clusters, in terms of rather complex probability functions, making possible certain useful predictions. In particular, provided the scale length of the distribution of particles was much less than the scale length of the turbulence (thus making it possible to apply the simplifications which arise within the inertial subrange of the spectrum of turbulence) solutions could be given for the rate of expansion at both ' small ' times and ' intermediate ' times. These solutions imply that a cluster of particles will grow initially at a rate $d\sigma/dt$ proportional to t (t = time, σ = particle standard deviation) and at a later time proportional to $t^{\frac{1}{2}}$. The constants of proportionality are, however, unknown. In testing these formulae of Batchelor, Gifford (1957a) collected smoke-puff data obtained by Frenkiel and Katz (1956), Kellogg (1956) and in a later paper (Gifford 1957b) data obtained by Tank. He concluded that the predicted forms of the expansive regime were satisfactorily confirmed by experiments. A cluster of particles, therefore, behaves very differently, in the earlier part of its expansion, from a continuous source plume, for which the rate of growth of any time-mean lateral

dimension is proportional to downwind distance (or equivalent time) raised to a power equal to, or less than, zero. However, since in any field of turbulence the statistical mean size of the cluster never exceeds that of the continuous plume, it follows that the rate of growth of the cluster must eventually behave in functionally the same way as the plume; that is, if locally $d\sigma/dt \propto t^{\alpha}$, then α, which is initially 1, must decrease through zero (constant rate of growth) to $-\frac{1}{2}$. This behaviour can be explained qualitatively by considering the action of a turbulent velocity field on a cluster of particles embedded in it. The energy of that field will be manifested in part by the motion of the centre of the cluster and in part by the motion of the individual particles relative to the centre. The growth of the cluster will be dependent on the proportion of energy in the relative motions. The larger the cluster becomes the greater will this proportion be until eventually the particles will be so far apart that they act independently. The growth will then be functionally equivalent to that of a continuous plume of serially-released independent particles, the motions of which express the full turbulent energy field.

The aim of this paper is to find quantitative expression for this picture and thereby determine formulae applicable to all sizes of cluster which express the rate of growth completely in terms of readily measurable properties of turbulence. A solution is found, under certain simplifying assumptions discussed below, which fully describes the whole expansion (Eq. (25)). It is shown that there exists a fairly extensive region in the expansion over which the rate of growth is approximately constant ($\alpha = 0$) and maximum and this is quantitatively described by a simply rule-of-thumb formula, Eq. (30), only involving the measurement of the total intensity of turbulence.

If the field of turbulence is represented by a continuous spectrum of eddies then the problem, both physical and theoretical, is to appreciate the nature and magnitude of the effect of these eddies on the cluster, taking into account the dependence of this process on both the eddy size and the cluster size (and therefore on time since the cluster is growing and the eddies are decaying and being replaced).

The inherent difficulties of both this problem and the complementary problem of the time-average distribution of particles from a continuous source have arisen mainly from the dependence of the diffusion process on the Lagrangian properties of the turbulence. To date, no satisfactory means of measuring these Lagrangian properties directly is available and so alternative methods have to be devised for attacking the problem. These, of course, must involve certain empirical assumptions. One such has been the analysis of Hay and Pasquill (1959) for the continuous source problem in which use has been made of the hypothesis that the Lagrangian (same particle) and Eulerian (fixed point) time-correlograms are similar in shape and differ only in the magnitudes of their time-scales. The method seemed to be encouragingly successful in that a wide range of appropriate diffusion results were shown to be consistent with a constant ratio β of these time-scales. It was decided therefore to attack the ' expanding-cluster ' problem in a similar fashion.

Starting with the basic equation representing the rate of change of the separation of a pair of particles in terms of relative velocities, the theory leads to prediction laws which are ultimately developed into a simple practical form giving the rate of growth of the cluster in terms of the mean-square fluctuation of wind direction. The effect of terminal velocity of the particles, if any, can be taken into account (see Section 5). Strictly the results relate only to homogeneous isotropic turbulence but they can be applied in a purely empirical sense to experiments carried out in inhomogeneous or anisotropic turbulence, such as that very near the ground.

Experiments carried out to test these formulae, in so far as this is possible, represent examples of diffusion over a wide range of conditions, from small-scale ground-level experiments over a few hundred metres to larger-scale free-atmosphere experiments over several kilometres. It should be noted that these experiments are limited in number and quality and are therefore partially inconclusive. However, as far as they go, they encourage the belief that here too β may be taken as a constant. This work is now presented because the accumulation of data for conclusive verification of the present theories is not likely within the foreseeable future.

2. Mathematical analysis

(a) *The assumptions and basic equations*

The spatial development of a cluster of ambient particles under the action of turbulence is complex but can be conveniently divided into the motion of the centre of gravity and the motion relative to the centre. Only the latter is investigated here. To predict this internal development completely would require an infinite amount of information and the power to analyse it. Consequently, statistical predictions are generally confined to the most important parameter, the root mean square separation of pairs of particles in the cluster. It is assumed that the turbulence is stationary, homogeneous and isotropic and that the form of the statistical number distribution of particles within the cluster is effectively constant. This distribution is taken to be Gaussian, in accordance with general practice.

The theory is developed to give the time-variation of the cluster size in terms of covariances of the velocity field. Generally, a velocity covariance is some mean product of velocities referred to their own mean values taken over specified positions in space and time relative to certain specified axes. This broad definition makes it desirable to clarify the usage adopted in this paper.

(i) Eulerian space covariances are mean products of velocities measured concurrently, at positions in space with specified separation.

(ii) Eulerian time- or auto-covariances are mean products of velocities measured at a fixed position in space and lagged by some specified time.

(iii) Lagrangian covariances are mean products of the velocity of a particle relative to the mean velocity of a group of particles taken at times with specified lag.

Put very simply, Lagrangian covariances are concerned with the motion in time of particles whilst Eulerian covariances are concerned with velocities at fixed points in space. But all these covariances are meaningless unless the extent of ' sampling ' and ' averaging ' are specified in terms of space and time. For example, the magnitude of the Eulerian space-covariance obtained from the time-trace of two wind vanes depends on the length of trace (i.e. the time of sampling) and the averaging interval over which the trace is read (including the automatic averaging engendered by the lack of response of the vane and recorders).

In the present problem the ' sampling ' is determined by the finiteness of the cluster; the covariances are defined in terms of the velocities of the particles relative to the mean velocity of all the particles in the cluster. This incomplete sampling of the turbulence is a reflection of the fact that the expansion is largely the result of eddies whose dimensions are roughly the same as, or smaller than, that of the cluster whilst the bigger eddies tend to shift the cluster as a whole bodily through space without contributing much to any internal distortion. ' Sampling ' and ' averaging ' can be thought of as high-pass and low-pass filtering of the full spectrum of turbulence. Mathematically, the problem is to determine this appropriately-filtered spectrum in terms of the output from one or more wind vanes. This is somewhat more difficult than the analogous task for the continuous source (Hay and Pasquill 1959) because here the characteristics of the high-pass filter (corresponding to the finite sampling) change as the cluster grows.

The analysis begins with the equation, given by Batchelor (1950), for the relative motion of a single pair of particles. If ξ_i is the vector separation of two particles, velocity u_{1i} and u_{2i}, then

$$\frac{d}{dt}\left[\xi_i(t) - \xi_i(0)\right] = u_{1i}(t) - u_{2i}(t) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Integration gives

$$\frac{d}{dt}\left[\xi_i(t) - \xi_i(0)\right]^2 = 2\int_0^t \left[u_{1i}(\tau) - u_{2i}(\tau)\right]\left[u_{1i}(t) - u_{2i}(t)\right] d\tau \quad . \quad (2)$$

Taking averages over all pairs of particles in the cluster and over all realizations of the turbulence, the left-hand side becomes

$$\frac{d}{dt}\overline{\xi_i^2(t)} - 2\frac{d}{dt}\overline{\xi_i(t)\,\xi_i(0)} + \frac{d}{dt}\overline{\xi_i^2(0)}$$

Now, $\xi_i^2(0)$ is not a function of t, and furthermore $\overline{\xi_i(0)\,d/dt\,\xi_i(t)}$ is zero because the subsequent velocity of any particle is independent of its initial position relative to any other particle. This leaves simply the first term. For a Gaussian cluster $\overline{\xi_i^2} = 6\sigma^2$ (σ is the standard deviation about the centre). Thus

$$\frac{d\sigma^2}{dt} = \tfrac{1}{3}\int_0^t \overline{[u_{1_i}(\tau) - u_{2_i}(\tau)]\,[u_{1_i}(t) - u_{2_i}(t)]}\,d\tau \quad . \quad . \quad (3)$$

If the velocities on the right-hand side are left as absolute velocities then, as Brier (1950) found, it is necessary to make some assumptions as to the nature of mixed Eulerian – Lagrangian correlations. In the context of the present analysis such assumptions will usually be too inaccurate for their use to be justified. It is far better to write the equation in terms of relative velocities when no such assumption is required :

$$u_i(t) = u_i'(t) + < u_i(t) > . \quad . \quad . \quad . \quad (4)$$

where $< u_i(t) >$ is the mean velocity of all the particles in the cluster at time t. Now $\overline{u'_{1_i}(t)\,u'_{2_i}(\tau)} = 0$ since keeping particle 1 fixed and averaging over particle 2 we have $\overline{u'_{1_i}(t)\,u'_{2_i}(\tau)}$ which is zero. Hence Eq. (3) becomes

$$\frac{d\sigma^2}{dt} = \tfrac{2}{3}\int_0^t \overline{u_i'(t)\,u_i'(\tau)}\,d\tau \quad . \quad . \quad . \quad . \quad (5)$$

This equation can now effectively be compared with Taylor's famous equation for the one-dimensional time-average dispersion of particles serially released from a continuous source in terms of the Lagrangian covariance :

$$\frac{d\bar{y}^2}{dt} = 2\int_0^t \overline{u(t)\,u(\tau)}\,d\tau$$

The fundamental difference lies in the meaning of the velocities. Here the growth of the cluster depends on a Lagrangian covariance expressed in terms of velocities relative to the mean velocity of all the particles, and therefore modified by the finite size of the cluster. For the continuous source, the full spectrum of turbulence is sampled; here, only a modified spectrum is sampled and one which changes as the cluster grows.

In the following paragraph and in the later mathematical treatments it is advantageously assumed that changes in Lagrangian time-scale τ_σ, appropriate to the cluster, resulting from changes in σ during a time $0\,(\tau_\sigma)$ are small compared with τ_σ itself (equivalently, $d\tau_\sigma/dt$ is small). The justification for this depends on the results of the analysis found later in the paper; and consists of finding the variation of τ_σ with σ for a typical spectrum of turbulence and using the experimental data in conjunction with the theory to find $d\sigma/dt$. From this it has been calculated that in these typical cases $d\tau_\sigma/dt$ has a maximum not exceeding 0·25 and is often much less than this. In the present context this value is deemed satisfactorily small. This assumption will be referred to subsequently as (A).

With (A) it follows that, when t is large compared with τ_σ, Eq. (5), as with Taylor's equation, can be written :

$$\frac{d\sigma^2}{dt} \approx 2\,\overline{u'^2_\sigma}\,\tau_\sigma$$

For large σ, both $\overline{u'^2}$ and τ become constants equal to those for the full spectrum. Then

$$\sigma^2 \approx 2\,\overline{u^2_\infty}\,\tau_\infty\,t$$

and so the change in σ over any period t is proportional to the intensity of turbulence $i = \sqrt{\overline{u^2}_\infty}/U$

$$\Delta\sigma \propto i$$

However, for smaller values of σ the variation of τ_σ and $\overline{u'^2_\sigma}$ with σ modifies this relation; giving

$$\Delta\sigma \propto i^2 \left[\frac{\overline{u'^2_\sigma}}{\overline{u^2}_\infty}\frac{\tau_\sigma}{\sigma}\right] = i^2\, k\,(\sigma), \text{ say.}$$

The change in σ with i depends on the functional form of $k\,(\sigma)$. For large σ, $k\,(\sigma)$ tends to zero like σ^{-1}. Near $\sigma = 0$, $k\,(\sigma)$ also tends to zero, since if the Eulerian trace-correlogram (see below for definition) is represented by $\frac{1}{3}R_{ii}\,(r)/\overline{u^2} = 1 - ar^\alpha$, ($a$ = some constant) for small r then by Eq. (5) of Smith's paper (1959) it follows that

$$\frac{\overline{u'^2_\sigma}}{\overline{u^2}_\infty} = b\,\sigma^\alpha \qquad b = \text{constant}$$

for small σ. Now because this form of the correlogram also implies that the energy spectrum at large wave-number behaves like

$$E\,(k) \propto k^{-(1+\alpha)}$$

we deduce that $\alpha > 1$ since the rate of energy dissipation is finite. Thus, with the additional fact that τ_σ also becomes zero at zero σ, it follows that $k\,(0) = 0$.

Consequently $k\,(\sigma)$ must have a turning point and, since $k\,(\sigma)$ is always non-negative, this turning point must be a maximum. In the vicinity of the maximum, then, $\Delta\sigma \propto i^2$.

This significant result states that during some part of the cluster's growth, the rate of expansion is so sensitive to the intensity of turbulence that it is proportional to the second power of i, instead of the first power as it is with very large clusters.

The analysis which follows may be considered as an attempt to define this relationship more explicitly. In Section 4 the experimental results will be inspected in the light of this prediction.

The next step, analogous to the Hay and Pasquill hypothesis, postulates that the Lagrangian time covariance and the Eulerian time covariance, both appropriate to the finite cluster of particles, are similar in shape and differ only in scale, the respective ratio of the scales being denoted by β. This can be interpreted in many ways according to which Eulerian time covariance, or related space covariance, is involved. (It is assumed that the space and time covariances are related by the Taylor transformation $r = Ut$).

Three possibilities are the longitudinal, the lateral and the trace covariances. The longitudinal space covariance is expressed in terms of velocity components along the direction of separation of the two points. The lateral covariance is in terms of components at right angles to the line of separation. The trace covariance is a combination of these two (one-third of the sum of the longitudinal and twice the lateral) whose main virtue is that in isotropic turbulence it is invariant to the angle between the line of separation of the two points and any of the fixed axes of measurement (the vertical, the downwind and acrosswind directions). The word ' trace ' is used because it is related to the sum of the diagonal elements (or trace) of the correlation tensor.

The length-scale of a covariance R is

$$l = \frac{1}{\overline{u^2}}\int_0^\infty R\,(r)\,dr$$

Thus for isotropic turbulence we have :

(i) the *longitudinal* covariance $\overline{u^2}\,f\,(r)$ with length-scale l, say, defining β_1

(ii) the *lateral* covariance $\overline{u^2}\,g\,(r)$ with length-scale $l/2$, say, defining $\beta_2 = 2\beta_1$

(iii) the *trace* covariance $R = \frac{1}{3}R_{ii} = \overline{u^2}\,(f + 2g)/3$ with length scale $\frac{2}{3}\,l$, say, defining $\beta_3 = \frac{3}{2}\,\beta_1$.

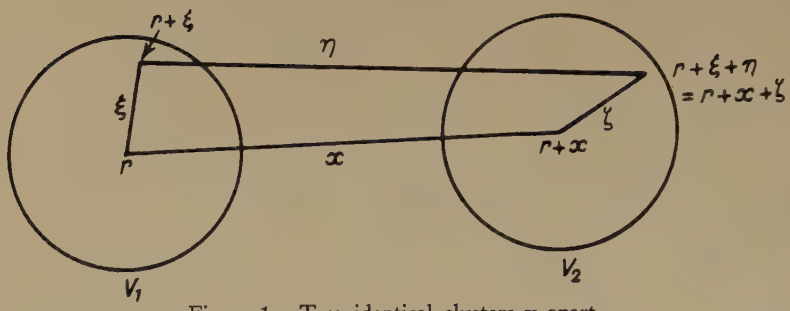

Figure 1. Two identical clusters **x** apart.

In their paper Hay and Pasquill used $g(r)$, defining a β_2, but perhaps the trace covariance is preferable because it not only simplifies the mathematical analysis but unlike the lateral covariance, can be non-negative at large r (there is no reason to suppose that the Lagrangian covariance ever becomes negative). Moreover the final solution is surprisingly insensitive to the choice of the Eulerian covariance. From now on the β will refer to β_3.

Eq. (5) becomes, on the basis of assumption (A) described previously,

$$\frac{d\sigma^2}{dt} = 2\int_0^t R'_L(t-\tau)\,d\tau = 2\int_0^t R_1\left(\frac{U}{\beta}(t-\tau)\right)d\tau \quad (R_1 \text{ is an Eulerian covariance})$$

$$= 2\frac{\beta}{U}\int_0^{Ut/\beta} R_1(s)\,ds \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the covariances refer to the modified field of turbulence experienced by the distributions. Consider this modified field of turbulence. Let two identical distributions, V_1 and V_2, of points in space have centres $\mathbf{r}$ and $\mathbf{r}+\mathbf{x}$ (see Fig. 1). Let the point $\mathbf{r}+\boldsymbol{\xi}$ belong to the first distribution V_1 and $\mathbf{r}+\boldsymbol{\xi}+\boldsymbol{\eta}$ belong to the second distribution V_2.

Define $$R(\boldsymbol{\eta}|\mathbf{x}) = \overline{u(\mathbf{r}+\boldsymbol{\xi})\,u(\mathbf{r}+\boldsymbol{\xi}+\boldsymbol{\eta})} \quad . \quad . \quad . \quad (7)$$

an average over all $\boldsymbol{\xi}$ in the distribution and over all realizations of the velocity field. Then with definitions given in Eq. (4) it is easy to show that

$$\overline{u^2} = R(0|0) = \overline{u'^2} + \overline{<u>^2} \quad . \quad . \quad . \quad (8)$$

$$R(x) = R(\mathbf{x}|\mathbf{x}) = \overline{u'(\mathbf{r}+\boldsymbol{\xi})u'(\mathbf{r}+\mathbf{x}+\boldsymbol{\xi})} + \overline{<u(\mathbf{r})><u(\mathbf{r}+\mathbf{x})>} = R_1(x) + R_2(x), \text{ say}. \quad (9)$$

$R_1(x)$ is the covariance between the velocities of particles belonging to identical distributions **x** apart, taken about the mean for each distribution, whilst $R_2(x)$ is the covariance between mean velocities of these same distributions. Therefore if $E(k)$, $E_1(k)$ and $E_2(k)$ are the appropriate energy spectra to $R(x)$, $R_1(x)$ and $R_2(x)$ respectively, then because E is a linear transform of R

$$E(k) = E_1(k) + E_2(k) \quad . \quad . \quad . \quad . \quad (10)$$

Now by Eq. (8) it is seen that the energy contributions from finite sampling and averaging are complementary. Furthermore $R_2(x)$ is unambiguously the appropriate covariance function for averaged velocities. Consequently $E_2(k)$ is unambiguously the appropriate spectrum function for averaged velocities. Hence $R_1(x)$ and $E_1(k)$ must be the appropriate covariance and spectrum functions for the finite-sampled velocities. It follows that $E_1(k)$ can be found for a Gaussian cluster of particles by first finding $E_2(k)$

$$R_{2ii}(\mathbf{x}) = \lim_{A\to\infty}\frac{1}{A}\int_A <u_i(\mathbf{r})>_{v_1} <u_i(\mathbf{r}+\mathbf{x})>_{v_2} d\mathbf{r} \quad . \quad . \quad (11)$$

where $$<u_i(\mathbf{r})>_{v_1} = \int_{v_1} u_i(\mathbf{r}+\boldsymbol{\xi})\,P(\boldsymbol{\xi})\,d\boldsymbol{\xi} \quad . \quad . \quad . \quad (12)$$

$P(\xi)$ is the Gaussian probability distribution function. Λ is a volume in the turbulent field enclosing V_1 and V_2. $\frac{1}{3}R_{2ii}$ is the trace covariance of type 2. Therefore

$$R_{2ii}(\mathbf{x}) = \lim_{\Lambda\to\infty}\frac{1}{\Lambda}\int_{\Lambda}\int_{v_1}\int_{v_2} u_i(\mathbf{r}+\xi)\,u_i(\mathbf{r}+\mathbf{x}+\zeta)\,P(\xi)\,P(\zeta)\,d\xi\,d\zeta\,dr$$
$$= \int_{v_1}\int_{v_2} R_{ii}(\mathbf{x}+\zeta-\xi)\,P(\xi)\,P(\zeta)\,d\xi\,d\zeta \qquad (13)$$

Writing $\eta = \zeta - \xi$, this can be simplified for a Gaussian cluster, standard deviation σ, to give

$$R_{2ii}(\mathbf{x}) = \frac{1}{8\pi^{3/2}\sigma^3}\int R_{ii}(\mathbf{x}+\eta)\,e^{-\eta^2/4\sigma^2}\,d\eta \quad . \quad . \quad . \quad (14)$$

an integral over all space. Writing $\mathbf{x}+\eta = \mathbf{r}$, integration over the sphere r = constant gives

$$R_{2ii}(\mathbf{x}) = \frac{1}{2\sqrt{\pi}\sigma}\int_0^\infty \frac{r}{x}R_{ii}(r)\left[\exp\left\{-\left[\frac{r-x}{2\sigma}\right]^2\right\} - \exp\left\{-\left[\frac{r+x}{2\sigma}\right]^2\right\}\right]dr \quad . \quad (15)$$

Now
$$R_{ii}(r) = 2\int_0^\infty E(k)\frac{\sin kr}{kr}\,dk \quad . \quad . \quad . \quad . \quad (16)$$

(see Batchelor 1953). Substituting this into Eq. (15) and integrating with respect to r finally gives

$$R_{2ii}(\mathbf{x}) = 2\int_0^\infty \{E(k)\,e^{-\sigma^2k^2}\}\frac{\sin kr}{kr}\,dk \quad . \quad . \quad . \quad (17)$$

which by Fourier's Inverse Transform theorem proves that

$$E_2(k) = E(k)\,e^{-\sigma^2k^2}$$

and
$$E_1(k) = E(k)(1-e^{-\sigma^2k^2}) \quad . \quad . \quad . \quad . \quad (18)$$

It is noted here that hypotheses alternative to that of Hay and Pasquill could be suggested. For example, Mickelsen has argued in favour of comparing space-correlograms rather than time-correlograms; the Lagrangian space-correlogram $R_L(s)$ being related to the Lagrangian time-correlogram $R_L(t)$ by the transformation $s = \sqrt{\overline{u^2}}\,t$. If the ratio of the length-scales is called b, then $b = \beta i$.

Clearly, b and β cannot both be absolute constants independent of i. However, the analysis that follows does not depend on the functional form of β on i, and this point may be deferred to the discussion of the results.

(b) *The spectral approach*

Finding $R_{1ii}(\mathbf{x})$ by substituting Eq. (16) and (17) into Eq. (9) and then putting this into Eq. (6) we obtain the equation for the expansion in terms of the full spectrum of turbulence

$$\frac{d\sigma}{dt} = \tfrac{2}{3}\frac{\beta}{U}\int_0^\infty\int_0^{Ut/\beta} E(k)\frac{\sin ks}{ks}\cdot\frac{1-e^{-\sigma^2k^2}}{\sigma}\,ds\,dk \quad . \quad . \quad (19)$$

where again assumption (A) has been implicity assumed. Now, provided Ut/β is large ($\gg\sigma$) it can be replaced in the integral limit by infinity. Since

$$\int_0^\infty \frac{\sin ks}{ks}\,ds = \frac{1}{k}\cdot\frac{\pi}{2} \quad . \quad . \quad . \quad . \quad (20)$$

It follows that

$$\frac{d\sigma}{dt} = \frac{\pi}{3}\frac{\beta}{U}\int_0^\infty E(k)\frac{1-e^{-\sigma^2k^2}}{\sigma k}\,dk \quad . \quad . \quad . \quad . \quad (21)$$

This suggests that it should be possible to measure the rate of expansion by passing an electrical signal proportional to the component of turbulence through suitable filters. In general it is easier to estimate the lateral wind component than the longitudinal component and therefore to investigate the spectrum of the lateral component $G(k)$ rather than that of the longitudinal component $F(k)$ or the true energy density $E(k)$. For isotropic turbulence

$$E(k) = k^3 \frac{d}{dk}\left[\frac{1}{k}\frac{dF}{dk}\right] \quad . \quad . \quad . \quad . \quad (22)$$

$$F(k) = k\int_0^\infty \frac{2}{\eta^2} G(\eta)\, d\eta \quad . \quad . \quad . \quad . \quad (23)$$

Substituting Eq. (22) into Eq. (21)

$$\frac{d\sigma}{dt} = \tfrac{4}{3}\pi \frac{\beta}{U}\int_0^\infty F(k)\{(2-\sigma^2 k^2)\,\sigma k\, e^{-\sigma^2 k^2}\}\, dk \quad . \quad . \quad (24)$$

and Eq. (23) into Eq. (24)

$$\frac{d\sigma}{dt} = \tfrac{8}{3}\pi \frac{\beta}{U}\int_0^\infty G(k)\, V(\sigma k)\, dk \text{ where } V(s) = \frac{1}{s^2}\int_0^s r^2(2-r^2)\, e^{-r^2}\, dr \quad . \quad (25)$$

The function $V(s)$ (plotted in Fig. 2) is the electrical window which could be designed to measure $d\sigma/dt$, for one value of σ, directly. So far no attempt has been made to do this, although similar filters used for finite-time sampling (Jones and Pasquill 1959) have been used successfully.

From Eq. (21) it is possible to derive a very useful practical result. Suppose the trace correlogram is exponential with length-scale l

$$R(r) = \overline{u^2}\, e^{-r/l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Then since

$$E(k) = \frac{3}{\pi}\int_0^\infty R(r)\, kr \sin kr\, dr \quad . \quad . \quad . \quad . \quad (27)$$

(Batchelor 1953), the spectrum is

$$E(k) = \frac{6}{\pi}\overline{u^2}\, l \frac{k^2 l^2}{(1+k^2 l^2)^2}$$

and

$$\frac{d\sigma}{dx} = 2\beta i^2 \int_0^\infty \frac{n^2}{(1+n^2)^2}\frac{1-e^{-r^2 n^2}}{nr}\, dn \quad . \quad . \quad . \quad (28)$$

where

$$i^2 = \frac{\overline{u^2}}{U^2}, \quad r = \frac{\sigma}{l}, \quad n = kl$$

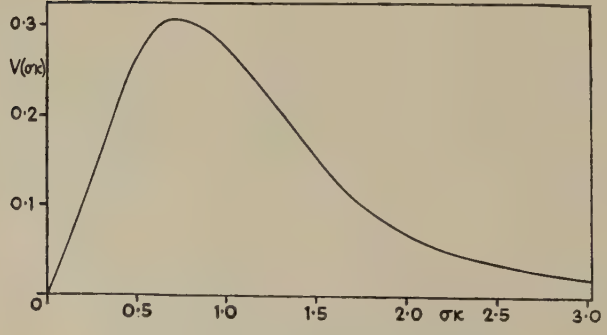

Figure 2. The filter characteristic $V(\sigma\kappa)$.

Now over a wide range of σ, the rate of expansion is almost constant and equal to the maximum value. This maximum value is given by

$$0 = \frac{d}{dr}\int_0^\infty \frac{n^2}{(1+n^2)^2}\cdot\frac{1-e^{-r^2n^2}}{nr}\,dn = \frac{1}{r^2}\int_0^\infty \frac{n}{(1+n^2)^2}\left[1-(1+2r^2n^2)\,e^{-r^2n^2}\right]dn \quad . \quad (29)$$

By numerical evaluation $r = 0{\cdot}655$. On substitution into Eq. (28), evaluation of the integral gives

$$\left[\frac{d\sigma}{dx}\right]_{\max} = \tfrac{2}{3}\,\beta_3\,i^2 \quad . \quad . \quad . \quad . \quad (30)$$

a result which is independent of the length-scale l. The suffix 3 is included as a reminder of the particular definition of β (above Eq. (6)). The only condition for (30) is that x/β should be large compared with σ. In the section discussing the experimental results it is shown that $\beta_3 \approx 4{\cdot}5$; so that $[d\sigma/dx]_{\max}$ is approximately equal to three times the variance of the fluctuations of a sensitive vane.

The first part of the cluster's growth, dependent on the initial distribution, is a relatively insignificant part of the whole for clusters which are initially small ($\sigma/l < 0{\cdot}1$) but which later grow large ($\sigma/l \geqslant 0$ (1)). For such, Eq. (21) gives the solution in terms of $E(k)$; and for an exponential correlogram this becomes Eq. (28) yielding a universal curve valid for any initially small cluster (see Fig. 3). Although generally the middle nearly-linear section dominates the curve, clusters which are initially very small ($\sigma/l < 0{\cdot}01$) have a significant initial sub-normal rate of growth which ideally should be taken into account. However, as in almost all of the present experiments the initial values of σ/l lie above this limit in a range $0{\cdot}01 < \sigma/l < 0{\cdot}1$, there is some justification for applying the simple rule-of-thumb formula (30). This saves the trouble of finding the length-scale l needed for the full Eq. (28). It should be noted that the solution for very small values of σ/l must depend rather more critically on the precise shape of the spectrum (or covariance) than for larger values of σ/l. Consequently, the use of filters conforming to Eq. (25) is required for a more reliable estimate of the cluster's behaviour at these very small values of σ/l.

(c) *The covariance approach*

Combining Eqs. (6) and (15)

$$\frac{d\sigma^2}{dt} = 2\,\frac{\beta}{U}\int_0^{Ut/\beta}\left\{R(s) - \sqrt{\frac{a}{\pi}}\int_0^\infty \frac{r}{s}R(r)\left[e^{-a(r-s)^2} - e^{-a(r+s)^2}\right]dr\right\}ds \quad . \quad (31)$$

where $a = \frac{1}{4}\sigma^2$. This has been evaluated for the exponential trace correlogram (Eq. (26)).

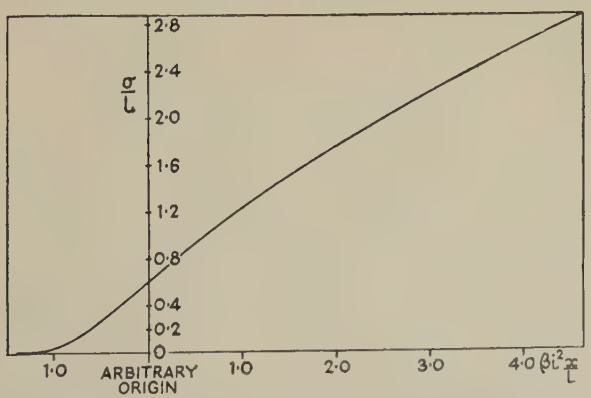

Figure 3. Universal expansion curve for initially small clusters.

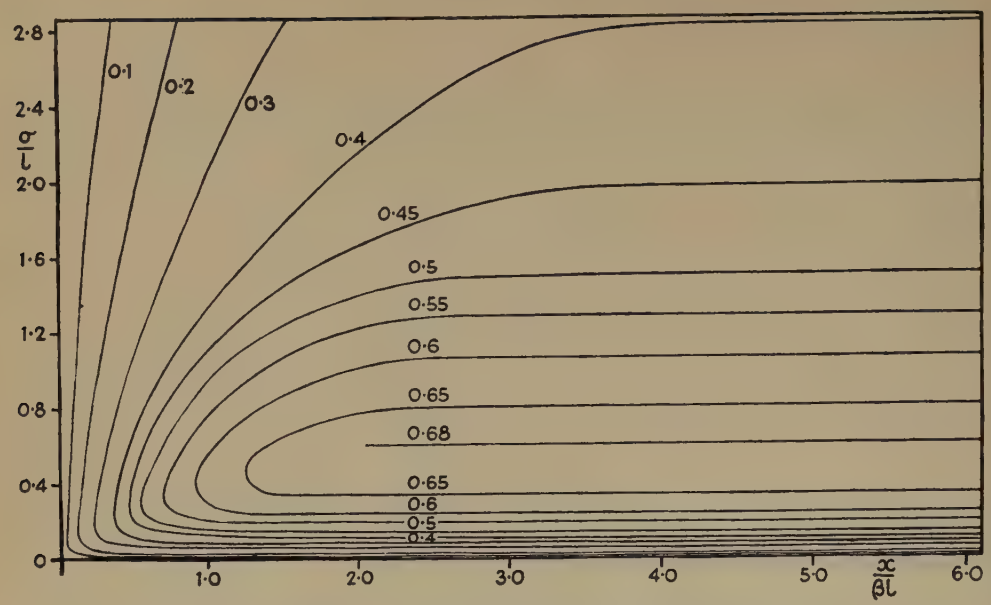

Figure 4. Contours of E where $\partial\sigma/\partial x = \beta i^2 E$.

Writing $x/\beta l = b$ and $\sigma/l = c$, Eq. (31) simplifies to

$$\frac{d\sigma}{dx} = \frac{\beta i^2}{c}\left[\operatorname{erfc}\frac{b}{2c} - e^{-b} - \tfrac{1}{2}e^{c^2}\left\{e^{b}\operatorname{erfc}\left(c+\frac{b}{2c}\right) - e^{-b}\operatorname{erfc}\left(c-\frac{b}{2c}\right)\right\} + z(b,c)\right] \quad . \quad (32)$$

where

$$z(b,c) = c^2 e^{c^2}\int_0^b \frac{1}{v}\left\{\operatorname{erfc}\left(c-\frac{v}{2c}\right).e^{-v} - \operatorname{erfc}\left(c+\frac{v}{2c}\right).e^{v}\right\}dv$$

This integral has to be evaluated numerically. Writing Eq. (32) in the following simple form

$$\frac{d\sigma}{dx} = \beta_3\, i^2\, E\left(\frac{\sigma}{l}, \frac{x}{\beta l}\right) \quad . \quad . \quad . \quad . \quad (33)$$

the function E is plotted in Fig. 4. The maximum value of E is 2/3 confirming the result obtained by the spectrum approach. The expansion curves can be calculated very easily from the result. Typical curves are shown in Fig. 5, in which the intensity of turbulence is taken as 0·1 and the length-scale of the trace correlogram as 15 metres, for various values of the parameter β.

3. Measurements of diffusion

(*a*) *Short-range experiments*

(i) Experimental technique

These experiments were carried out over downland, with grass 10-15 cm long, and each involved the release at a height of about 2 m of 20 gm of Lycopodium spores. By catapulting the material from a small container any tendency for bulk settling was avoided and within a few metres of travel the spores were found to be dispersed into a cluster approximately 1 to 2 m in height and width and 2 to 4 m in length. This was taken to be the initial stage of the cluster, subsequent diffusion was assumed to be due only to natural turbulence.

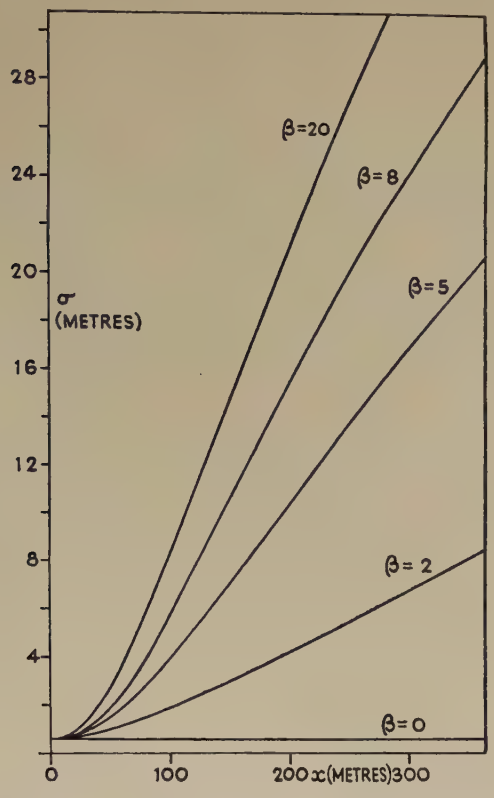

Figure 5. Typical expansion curves for five values of β; $l = 15$ m; $i^2 = 0{\cdot}02$; $\sigma(0) = 0{\cdot}6$ m.

The particles were collected by natural impaction on adhesive cylinders as in continuous-source experiments described by Hay and Pasquill (1959). In the first six experiments, 29 cylinders were set out on each of three crosswind lines 100, 200 and 300 m downwind, the spacing in these lines being 2, 4 and 6 m respectively. The cylinders were mounted at a height of about 60 cm on wooden posts. In four later experiments, only the 100 m line was used.

A continuous measurement of wind speed and direction at a height of 2 m was provided by an anemometer and a light wind vane set up close to the dispenser. Additional wind-direction measurements at 2 m were provided by three similar vanes positioned crosswind, the separations ranging from 5 to 35 m. The measurements were displayed on pen recorders running at a speed of 3 in min^{-1}.

(ii) Analysis of observations

The particles on twenty 0·15 cm wide sections of each sampling cylinder were counted under a microscope and the mean number per section taken as a measure of the total deposit on the cylinder during the passage of the cluster. The apparent width of a cluster obtained by this method is a good estimate of its true width at the time of passage. Such small differences, as may exist between the apparent and true widths, arise largely from the cluster having a component of motion along the line of sampling cylinders. This motion, which is a result of the turbulence can either persist or may fluctuate in magnitude as the cloud passes the samplers. An estimate of its average effect on the width can be obtained from the measured intensity of turbulence, since if θ is the angle between the cluster's motion and the line at right-angles to the sampling line, then

$$\sigma_{app.} \simeq \sigma_{true} \sec\theta$$

On averaging

$$\bar{\sigma}_{app.} \simeq \bar{\sigma}_{true}\left(1 + \frac{i^2}{2}\right)$$

which implies a very small mean error of at most 1 or 2 per cent in the experiments reported below. It follows that the type of sampling gives reliable estimates of the instantaneous width, differing considerably from the width of the continuous source plume equivalent to the superposition of many clusters whose centres are themselves dispersed along the sampling line as a result of the large-scale turbulent motions.

As an example of the results obtained, the crosswind distributions in experiment No. 4 are shown in Fig. 6. The computed standard deviations σ of the crosswind spreads of particles for all experiments are given in Table 1. Where no value is quoted, the particle counts were so low that a reliable standard deviation could not be derived.

TABLE 1. RESULTS OF SHORT-RANGE EXPERIMENTS

Experiment No.	Date	Time GMT	Standard deviation (m) of particles at 100 m	200 m	300 m	Standard deviation of wind direction* radians	mean wind speed* m sec^{-1}
	1958						
1	20 June	1137	4·76	10·80	17·70	0·136	6·1
2	20 June	1149	3·16	7·64	–	0·153	5·6
3	27 June	1348	3·68	6·80	13·98	0·151	6·1
4	27 June	1447	5·04	10·28	14·40	0·126	5·0
5	27 June	1458	5·38	11·92	–	0·147	4·3
6	27 June	1509	–	5·80	9·24	0·140	5·9
	1959						
7	17 March	1500	3·48	–	–	0·113	9·5
8	17 March	1518	4·70	–	–	0·085	9·1
9	17 March	1525	2·20	–	–	0·091	9·7
10	17 March	1534	5·37	–	–	0·095	9·0

* These data refer to a height of 2 m and a sampling time of 3 min.

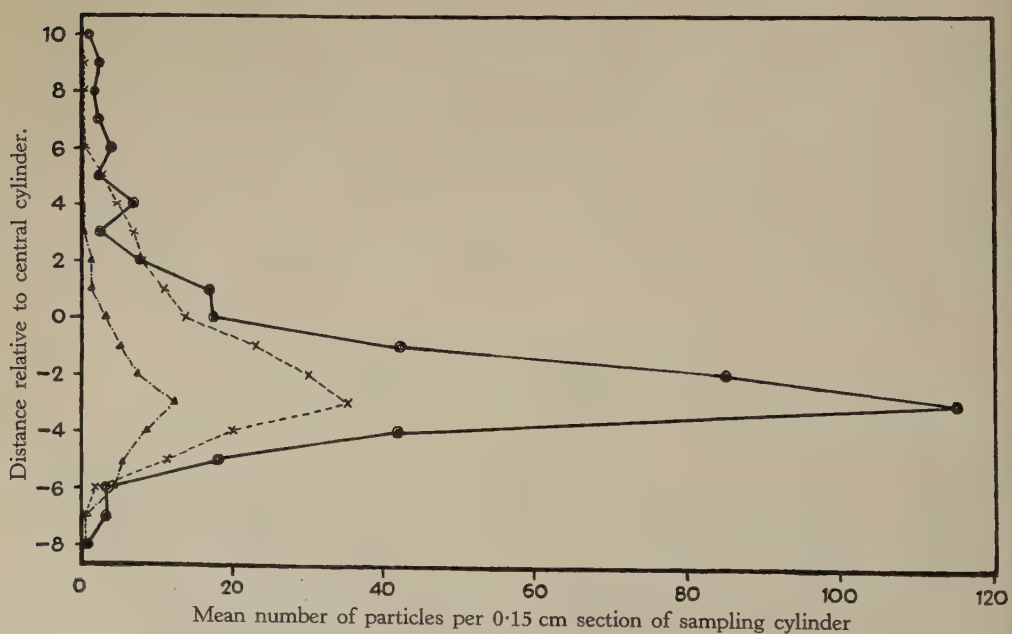

Figure 6. Crosswind distributions of particles in a cluster after 100 m (— ⊙ —); 200 m (– – × – –); 300 m (· – △ – ·) travel in short-range experiment No. 4. Ordinate scales are 1 unit = 2, 4 and 6 metres respectively.

From each of the wind records consecutive one-second average values were extracted for a period of 3 min centred on the time of release. Standard deviations were computed and also the correlation coefficients $R(\theta, y)$ between wind directions θ at crosswind points separated by distance y. The four values of standard deviation of direction $(\overline{\theta'^2})^{\frac{1}{2}}$ obtained in each experiment were usually within 15 per cent of the mean and, in the subsequent discussion, only the value measured closest to the dispenser is employed. These values given in Table 1 represent a close approximation to the intensity of turbulence $i = (\overline{v^2})^{\frac{1}{2}}/U$, where v and U have their usual significance.

Values of the length-scale l were estimated from the areas under the correlograms $R(\theta, y)$ from $y = 0$ to $y = 35$ m, the maximum separation of the vanes. In such instances, when $R(\theta, y)$ was still significantly non-zero at 35 m another estimate of length-scale l could be derived by fitting an exponential curve of the form $R(\theta, y) = e^{-y/l}$, a procedure justified by the quasi-exponential form of most of the correlograms. The values of l obtained in these ways ranged from 5 to 25 m and did not appear to vary systematically with any meteorological factor.

In the preceding mathematical treatment, the working formulae involve the intensity of turbulence and so the period of record, whilst for practical reasons kept as short as possible, must be at least an order of magnitude greater than the time scale of the turbulence. In the present experiments, the ratio of the former to the latter time is at least 30.

Furthermore the intensity i should include the turbulent energy at all high frequencies whereas the method of reading the wind traces results in a loss of 50 per cent of energy associated with a frequency of $\frac{1}{2}$ c sec^{-1} and progressively greater loss at higher frequencies. A theoretical estimate of the total energy lost may be made using the assumed exponential form for the correlogram and in the very worst case this amounts to an underestimate in i of 15 per cent.

(*b*) *Medium range experiments*

(i) Experimental technique

In these experiments, zinc cadmium sulphide particles, which fluoresce a characteristic yellow when irradiated with ultra-violet light, were employed as the tracer element. They were released from an aircraft flying on a crosswind track, the centre of which was several miles upwind of sampling apparatus mounted on the cable of a captive balloon. The choice of height of release, which ranged from 1,500 to 2,500 ft, was principally dependent on the height of the cloud base while the upwind distance was chosen after an inspection of the initial wind observations, so that the particles should barely reach the top and bottom sampling points. On each occasion, three or four releases were made in succession, with a suitable time interval between to ensure that the particle clouds did not merge before they reached the balloon.

The sampling apparatus was located on the ground and at six points on the balloon cable, usually at heights of 1,000, 1,500, 2,000, 2,500, 3,000 and 4,000 ft. The balloon-borne sampling units consisted of drum impactors designed to give up to 104 samples over successive one-minute intervals. The flow rate of each sampling unit was measured on the ground before and after each series of experiments.

During the experiments, measurements were made of wind speed and inclination by means of three vertical gustiness meters (Jones and Butler 1958) mounted on the balloon cable, usually at heights of 1,000, 2,000 and 3,000 ft. The outputs from these instruments were brought to the ground by lightweight electric cables and the inclination signals fed into electrical filters of the type described by Jones and Pasquill (1959). The filters separated the wind inclination fluctuations into low- and high- frequency components; the low-pass outputs were recorded directly whilst the high-pass outputs were rectified and presented as equivalent standard deviations. Wind directions used in predetermining the release track of the aircraft were noted from pennants attached to the balloon cable. Occasional theodolite observations of the elevations of these pennants were used to convert the slant heights previously quoted to true heights.

(ii) Analysis of observations

The drums were examined under a microscope, with ultra-violet illumination, and the number of fluorescent particles on each exposed section was counted. These were summed over the time of passage of each cloud and then normalized to a standard flow rate of 20 litres/minute. By the same argument as advanced in 3 (a) (i) the distribution obtained in this way may be taken as a good approximation to the instantaneous distribution in the vertical plane.

In only one experiment was the ideal type of vertical distribution realized, and this is shown in Fig. 7. In some cases, vigorous diffusion in the lower layers of the atmosphere resulted in particle collections near the ground being greater than aloft, whilst in others the diffusion was so slow that particles were collected at only one or two points. Such complications are well illustrated by the experiments on 31 October 1958, (see Table 2), when the release was at 2,500 ft. Initially, the turbulence was quite pronounced at 1,000 ft but very slight at 2,000 and 3,000 ft, and it was clear that in experiments 8 and 9, the particle clouds were entirely in the region of low turbulence. The turbulence at 2,000 and 3,000 ft then suddenly started to increase but the next cloud, arriving at 2,500 ft, was still shallow. By the time of the final release (experiment 11), however, the intense turbulence had evidently extended to the level of the cloud as particles were now diffused downwards to the ground.

In interpreting the numbers of particles collected, any variation of wind speed with height should ideally be taken into account. Now such variations are generally significant only in the surface layers, but if the cloud has substantially reached the ground, it is clear that in any case the standard deviation σ of the particles can be estimated only approximately. Moreover, the theoretical analysis of diffusion does not account for any such variations. For these reasons, the normalized numbers of particles collected are used without further adjustment. In some cases the shapes of the vertical distribution of particles were such that only upper and lower limits of σ could be estimated. Values of σ, including some from previous experiments involving travel over 50 miles are given in Table 2.

The wind-inclination data is in two parts, the first representing the time variation of the equivalent standard deviation corresponding to a length of record of 30 sec and the second the time lapse variation of inclination averaged over 30 sec. From these may be obtained the standard deviation of inclination for any length of record greater than 30 sec. As these records indicated significant eddies with periods up to six minutes, the required total intensity, for effectively infinite sampling time, was derived from a 30 min (or longer) length of record which included the time of passage of successive clouds. On 31 October 1958, as the turbulence increased suddenly during sampling the data have been divided into periods before and after this change. Standard deviations of inclination and mean wind speed at each level are included in Table 2.

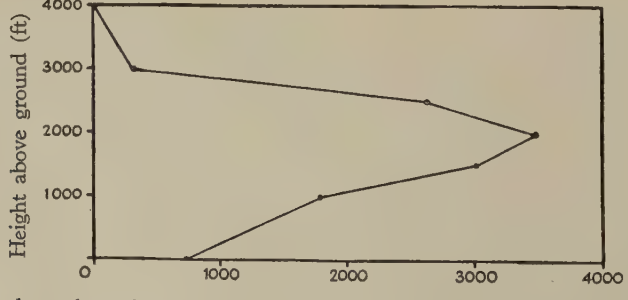

Figure 7. Vertical distribution of particles in medium-range experiment No. 16.

TABLE 2. RESULTS OF MEDIUM RANGE EXPERIMENTS

Expt. No.	Date	Time GMT	Height of release ft	Distance of release miles	σ ft	Standard deviation (radians) of wind inclination at nominal heights of 1,000 ft	2,000 ft	3,000 ft	Mean wind speed ($m\ sec^{-1}$) at nominal heights of 1,000 ft	2,000 ft	3,000 ft
	1958										
1	23 Sept.	1055	2,000	2·9	725–795	0·093	0·082	0·065	8·3	8·9	9·6
3	23 Sept.	1120	2,000	8·1	1,120–1,460						
5	26 Sept.	1025	2,000	2·2	1,270	0·204	0·194	0·157	4·9	5·3	5·7
8, 9, 10	31 Oct. 31 Oct.	1035–1100	2,500	9·3	80–170	0·080	0·025	0·020	7·6	–	9·5
11	31 Oct.	1115	2,500	8·3	280–350	0·104	0·071	0·045	7·0	–	9·3
	1959										
12, 13	4 Mar.	1010–1025	1,500	2·7	160–315	0·060 (500)	0·028 (1,500)	0·022 (2,500)	11·3 (500)	14·1 (1,500)	14·5 (2,500)
15	7 May	1015	2,500	2·0	200–450	–	0·159	0·122	5·4	5·3	4·7
16	7 May	1023	2,500	2·0	500						
17	7 May	1030	2,500	2·0	250–500						
18	7 May	1037	2,500	2·0	400–900						
	1957										
B	14 Nov.	1315–1415	1,000	63	1,500–?	0·076 (400)	0·054 (2,200)	–	9·0 (400)	11·3 (2,200)	–
C	15 Nov.	1500–1700	1,000	44	230–450	0·015	–	–	5·7	–	–

Figures in brackets are heights of measurements in feet when different from those quoted at heads of columns.

The amount of high-frequency energy lost is determined by the response characteristics of the vane, and is approximately 50 per cent at frequencies of 2 cycles sec^{-1} (equivalent to a quarter-second averaging interval) and greater at higher frequencies. In view of the rapid decrease of energy with increasing frequency at this stage of the spectrum, it is obvious that the total loss is even less important than for the short-range experiments.

The filter circuits referred to above had a 50 per cent energy cut-off frequency of 0·88 cycles min^{-1}. In addition, there were two other high-pass filter circuits with 50 per cent cut-off frequencies of 5·3 and 0·147 cycles min^{-1}, to which the inclination signal from the central instrument was applied. The outputs from both these circuits were likewise recorded directly as equivalent standard deviations. All these data were used to derive estimates of the turbulent energy $G(n)$ within certain frequency bands of the spectrum. Values of $nG(n)$ were then plotted against $\log n$, and the frequency at which the resulting curve had a maximum gave the desired rough estimate of the length-scale l_3 of the turbulence by means of the formula

$$l_3 = \frac{0{\cdot}078\,U}{n_{\max}}$$

derived on the assumption of an exponential trace correlogram.

When the data from six of the trials were treated in this way, in all but one case the length-scales so obtained lay between 270 ft and 1,600 ft. This result seems to be consistent with figures, obtained by more detailed analysis, given by other workers. Press and Tukey (1956) state that their measurements of spectra, using aircraft, give a representative

value of length-scale of the order of 1,000 ft. Similarly, Panofsky and Deland (1959) have given an empirical formula for length-scale as a function of height based on data obtained mainly (but not exclusively) from below 300 ft which if applied at a height of 2,000 ft gives a length-scale approximately equal to 500 ft.

4. Discussion of results

One of the conditions of validity of Eq. (30) is that the standard deviation of the cloud should be within an order of magnitude of the length-scale of the turbulence. As already indicated, rough estimates of the scale were made for both the short-range and medium-range experiments. In the former case the scales were in the range 5-25 m; in the latter case they were of the order of 300 m, and so in both cases the size of the cluster was commensurate with the scale. Further confirmation that the experimental conditions approximated to those under which Eq. (30) was obtained comes from the variation of σ with distance in the short-range experiments. The mean rate of expansion of the cluster from the source to the three sampling lines showed only a small systematic variation with distance – the rate of growth was approximately constant. It has been noted already that the full theory (Eq. (33)) predicts that, after the initial acceleration of the rate of growth in accord with Batchelor's dimensional arguments mentioned in the Introduction, the cloud settles down to a fairly extensive region in which the rate of growth is approximately constant. Examination of experiments 1-6 in Table 1 shows that the ratio of σ (200 m) to σ (100 m) is greater than the ' linear growth ' value of 2·0 by 7 per cent on the average, and the ratio of σ (300 m) to σ (200 m) greater than 1·5 by 12 per cent. Whilst the number of experiments render these figures rather uncertain, the indication of slight positive acceleration is in accord with the full theory. Nevertheless, the departure from linear growth at this stage of the cluster's development is, if significant, clearly quite small and it seems reasonable therefore to apply Eq. (30) in the finite-difference form to the present experiments :

$$\frac{\Delta\sigma}{\Delta x} = \tfrac{2}{3}\beta i^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

One point which had to be decided was whether or not it was reasonable to consider all the results together since they are rather heterogeneous, including as they do values corresponding to a thousand-fold range of cluster size, a ten-fold range in intensity of turbulence, to ground-level experiments in which wind shear may play an important role and to experiments in which anisotropy is introduced by the presence of free convection. However, the amount of data does not really justify any division on these lines even if it were possible to do so. Moreover if it can be shown that Eq. (34), applied empirically, has universal validity with one value of β equally appropriate to all these different conditions then it would indeed be a most valuable formula. With these points in mind it was decided to present the data all together.

The results have to be investigated to see whether, in the light of Eq. (34), there is sufficient mutual consistency to indicate the existence of a β and if so, to determine the functional dependence of β on the intensity i. For convenience in the presentation of the data, Fig. 8 shows a plot of $\sqrt{\Delta\sigma/\Delta x}$ against i instead of $\Delta\sigma/\Delta x$ against i^2. In the case of the short-range experiments, increments in standard deviation $\Delta\sigma$ for $\Delta x = 100$ m were determined from the values given in Table 1. In determining the average rate of growth up to the 100 m sampling-line an estimate of the initial size of the cluster was needed. Remembering that the initial growth of the cluster was not unaffected by the mode of release, a virtual σ, appropriate to $x = 0$, was deduced from visual observations made shortly after release and the measured spread at 100 m. Crude though this procedure is, the values of σ (0) are so small that errors arising from taking a single representative value of 10 cm are believed to be unimportant.

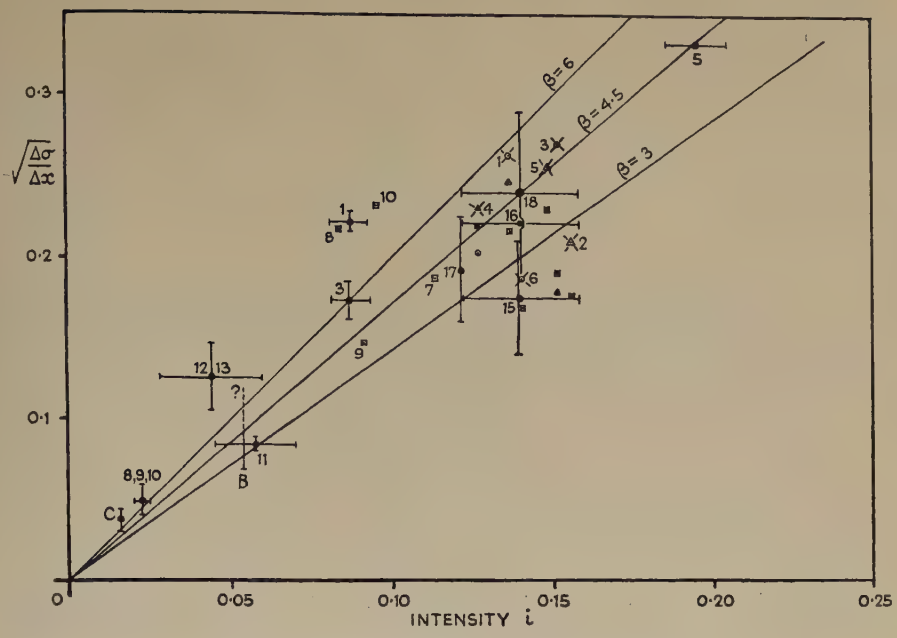

Figure 8. Results of the diffusion experiments.

KEY

● Medium and long-range experiments

Short-range experiments :

⊡ based on travel between 0-100 metres
△ based on travel between 100-200 metres
⊙ based on travel between 200-300 metres
⋇ denotes the maximum for that experiment

In the medium-range experiments, σ was taken as zero initially and values of $\Delta\sigma/\Delta x$ were derived by dividing the values of σ quoted in Table 2 by the corresponding upwind distance of the emission track from the balloon. Where a range of possible values of $\Delta\sigma$ is given in Table 2, this is represented by a line in Fig. 8. Again, if the diffusion of the cloud could not be associated with the turbulent intensity at one specific height, the appropriate range of intensity is indicated.

With regard to the aim stated above, it will be seen that, in general, the points on the figure follow a linear trend; that is, β is independent of intensity. Eq. (34) is represented on the figure by three lines for which $\beta = 3$, 4·5 and 6 and of these, $\beta = 4{\cdot}5$ clearly gives the best overall fit. With this limited amount of data a more detailed analysis than this would not be justified. A considerable scatter must be admitted, but considering the large range of conditions covered, it does not seem excessive. Apart from four rather exceptional results, the points lie within about 30 per cent of the line $\beta = 4{\cdot}5$ for any particular value of the intensity. Of the four exceptional results referred to, two were obtained in short-range and two in medium-range experiments. In the former (experiments 8 and 10), the crosswind distributions of particles were far from Gaussian and had in fact more than one peak, suggesting that the clusters may have been subjected to unusual distortions in the early stages of their travel. Of the latter, the result of the trial on the 4 March 1959 remains a definite anomaly. The particles released at about 1,500 ft reached the ground in fair numbers in just under three miles travel. To do this the cloud would have to experience what would be equivalent to a constant downward fall of approximately 1 in 10. Such a prolonged descent of the air would be expected to show up on the low-frequency traces, but there is no sign of any such descent and the only reasonable conclusion reached

is that there must have been a marked lack of homogeneity in the field of turbulence over the region traversed by the cloud. The fourth case (experiment 1) was another very uneven distribution which may have been affected by the convection that was present.

With the above qualifications always in mind, the general conclusion to be drawn from this analysis is that a useful estimate of the spread of a cloud of particles can be made by the equation

$$\frac{\Delta\sigma}{\Delta x} = 3i^2$$

Since the necessary type of measurement of i can now be made, this method can profitably be used, either in specific experimental studies or in the statistical estimation of certain appropriate properties of clouds of airborne material.

5. Falling clusters

In many practical problems the terminal velocity of the particles cannot reasonably be taken as zero. Smith (1959) has discussed this case and given a solution valid when two conditions are concurrently fulfilled. These conditions are that the terminal velocity should be great enough for the effect of Lagrangian decay of the eddies on the expansion to be neglected, whilst small enough for the particles' response to the velocity fluctuations affecting the expansion to be almost perfect.

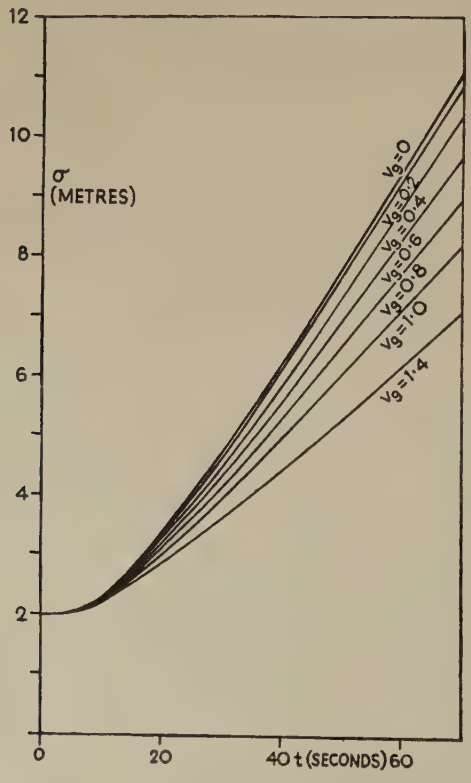

Figure 9. Expansion curves for clusters of particles with terminal velocity v_g m sec^{-1}, $v = 5$ m sec^{-1}, $l = 20$ m, $\beta = 5$, $i = 0{\cdot}1$, $\sigma(0) = 2$ m.

Now the method used in the present analysis for the floating cluster gives an equation of expansion very similar to that for the falling cluster in the solution mentioned.

Floating cluster : $\frac{d\sigma^2}{dt} = \frac{2}{3}\frac{\beta}{U}\int_0^{Ut/\beta} \{R_{ii}\}\, ds$ where the integrand is a function of R_{ii} – see Eq. (31) in which $R(s) = \frac{1}{3} R_{ii}(s)$.

Falling cluster : $\frac{d\sigma^2}{dt} = \frac{2}{3}\frac{1}{v_g}\int_0^{v_g t} \{R_{ii}\}\, ds$ with the same integrand. v_g is the terminal velocity.

Thus one transforms into the other if v_g is replaced by U/β (or vice-versa). Physically this means that horizontal travel at a velocity U is equivalent, as far as cluster development is concerned, to a vertical motion at a velocity U/β. Therefore if the horizontal scale is compressed by a factor β, in these transformed coordinates the direction of the motion is irrelevant, the cluster experiences the same ' statistics.' And so, in the general case, when neither U/β or v_g can be neglected with respect to the other, the real ' velocity ' is not v_g or U/β but $\sqrt{v_g^2 + U^2/\beta^2}$.

Thus $$\frac{d\sigma^2}{dt} = \frac{2}{3}\frac{1}{\sqrt{v_g^2 + U^2/\beta^2}}\int_0^{t\sqrt{v_g^2 + U^2/\beta^2}} \{R_{ii}\}\, ds$$

which, of course, reduces to the special cases when either U/β or v_g becomes dominant. The solution, when an exponential trace correlogram is taken, is thus

$$\frac{d\sigma}{dt} = \frac{\overline{u^2}}{\sqrt{v_g^2 + U^2/\beta^2}}\, E\left(\frac{\sigma}{l}, t\sqrt{v_g^2 + U^2/\beta^2}\right),$$

the function E being the same as before (see Fig. 3). Typical solutions are given in Fig. 9 which show the effect of increasing the terminal velocity on the growth of the cluster.

Acknowledgments

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The meteorological significance of remote measurements of infra-red emission from atmospheric carbon dioxide

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SUMMARY

Observation from outside the atmosphere of the emission from different wavelength regions in the 15μ carbon-dioxide band, would give information of the vertical temperature distribution between 10 and 50 km altitude. The measurements required are feasible using a Fabry-Perot interferometer, and sample calculations of the depth in the atmosphere of the origin of this emission are presented.

1. INTRODUCTION

It has been suggested by Kaplan (1959) that the temperature structure of the upper atmosphere may be inferred by measuring from above the emission from different wavelength intervals in the 15 μ band of carbon-dioxide. Such observations could be made from an artificial satellite, with the possibility of measuring the vertical temperature distribution over large areas of the earth's surface over long periods of time. Kaplan suggested that an infra-red spectrometer having a band-width of 5 cm^{-1} would be sufficient to provide a coarse resolution of the details of the vertical temperature structure, but points out that the interpretation of such measurements would be complex. A method is here suggested which would make more efficient use of the infra-red energy available, and for which interpretation will not be so difficult.

2. INSTRUMENTATION

The rotational lines in the short-wavelength half of the 15 μ band are shown in Fig. 1. The line positions and intensities for the P and R branches are taken from Yamamoto and Sasamori (1958), and for the Q branch from Kaplan (1947). In order to select radiation from a small depth of atmosphere, a portion of the band having as uniform an absorption coefficient as possible must be isolated. Because of the very regular structure of the lines of the $00^{\circ}\ 0\text{-}01^{1}0$ band, the most efficient way in which the radiation can be selected is to use an interferometer having a spacing between orders equal to the spacing between the lines of the band. This is approximately 1·6 cm^{-1}, and could be matched by a Fabry-Perot etalon having an air gap of about 3 mm. Such an interferometer would act as a ' comb filter,' and a change in optical path of about 0·1 per cent would move the positions of the maxima of transmission from the line centres to symmetrically between the lines. At the line centres emission will be from high in the atmosphere, whereas between the lines the emission will originate from relatively lower levels.

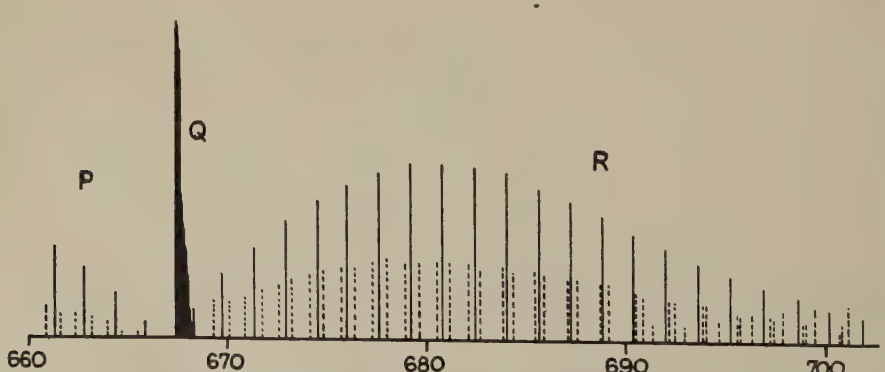

Figure 1. Positions and relative strengths of the lines in carbon-dioxide bands near 15 μ. Full lines are lines of the $00^{\circ}0\text{-}01^{1}0$ band centred at 667·40 cm^{-1}. Dotted lines, the heights of which correspond to line strength multiplied by 10, belong to the $01^{1}0\text{-}10^{\circ}0$ or the $01^{1}0\text{-}02^{2}0$ bands. Other bands, which are considerably weaker, have been ignored.

An interferometer for use at a wavelength of 2 μ, having a finesse (ratio of half-width of pass-band to spacing between orders) of 30 has been described by Smith and Heavens (1957). Dr. S. D. Smith (private communication) considers it entirely possible that a similar performance could be obtained at 15 μ. If, however, the pass-band is too narrow, insufficient energy will reach the detector. In this study, the interferometer has been assumed to have a finesse of 16 or a half-width of pass band of 0·1 cm^{-1}. Such an interferometer would be used with a broader band interference filter having a band-pass of about 24 cm^{-1}. Again such have been constructed for the near infra-red (see Heavens, Ring and Smith (1957)), and research on filters for longer wavelengths is being actively pursued. In this study, it has been assumed that the region 669·7-693·7 cm^{-1}, which includes 15 lines of the $00^{0}0\text{-}01^{1}0$ band, has been isolated in this way.

The total band-width accepted by such a system will be about 1·5 cm^{-1}. A detector of 1 mm^2 in area, receiving energy in this band-width from a solid angle of 0·0025 (equivalent to a cone of angle 5°) from a large depth of carbon-dioxide at a temperature of 225°K, would receive 2×10^{-10} watts. If this is to be measured to 5 per cent, a detector having a minimum detectable energy of 10^{-11} watts is required. An ideal thermal detector of this area at 300°K and operating with an amplifier band-width of 1 c/s can detect about 5×10^{-12} watts. An ideal quantum detector of this area, but insensitive to wavelengths beyond 15 μ would be a factor of two better than this. These ideal limits vary as $T^{5/2}$ where T is the temperature of the detector and its environment, and so can be reduced by working with the detector at a low temperature. Morton, Schultz and Harty (1959) have described photoconductors constructed from mixed crystals of germanium and silicon with zinc as an impurity, and operated at a temperature of about 50°K. The minimum detectable energy for one of their detectors of 1 mm^2 area is about 10^{-11} watts. This is only just sufficient. Some gain would, however, result from using a detector of larger area.

Cooling of the detector to a low temperature could be provided in a satellite by using a large reservoir of liquefied gas. If this were hydrogen, such a reservoir would last a few weeks at the most. The most attractive way to provide permanent cooling is to arrange for the detector to lose heat by radiation to space. Since, however, total radiation varies as the fourth-power of the temperature, the detector would have to be isolated so as to receive less than 0·1 per cent of possible radiation from its surroundings, in order to reduce its temperature by a factor of 6. Other ways of refrigerating the detector would be to construct a miniature liquefier or to use thermoelectric cooling. The former, however, would have embarrassing power requirements, and it is doubtful whether the latter could provide more than a few degrees of cooling at low temperatures. Providing permanent cooling to below 50°K will probably prove the greatest technical hurdle to using sensitive infra-red detectors in satellites.

3. Interpretation

Some calculations have been made of the depth in the atmosphere from which the radiation transmitted by such an interferometer would originate. The results of the calculations, which were made using the Oxford Mercury Computer, are shown in Fig. 2. The fractional transmission $\tau(z)$ between a height z and the top of the atmosphere has been calculated for the wavelength region passed by the interferometer, for values of z differing by 5 km. The incremental transmission $\tau(z_1) - \tau(z_2)$ for 5 km layers, i.e. for $z_1 - z_2 = 5$ km, has been plotted against $\frac{1}{2}(z_1 + z_2)$ in Fig. 2 (a). The following assumptions were made :

(1) That the mixing-ratio of carbon-dioxide in the atmosphere is constant with height and equal to 3×10^{-4}.

(2) That the spacing between orders of the interferometer is 1·59 cm^{-1} and its transmission bands are rectangles 0·1 cm^{-1} in width. The spacing between the lines in the wavelength region 669·7-693·7 cm^{-1} varies between 1·57 cm^{-1} and 1·61 cm^{-1}. The upper curve in Fig. 2 (a) has been constructed for the case when the maxima of transmission of the interferometer coincide with the line centres in the centre of the region.

(3) That the lines are of Lorentz shape having a half-width of 0·17 cm^{-1} at STP (Kaplan 1950). Doppler broadening, which will become significant in the region very close to the line centre at pressures below 1 mb, has been ignored. Allowing for this would make a small change in the shape of the upper curves in Fig. 2 (a).

(4) That the effect of temperature on half-width and line strength is negligible.

(5) That the effective half-width for a path of absorber between a level at pressure p and the top of the atmosphere is the same as that for a path of uniform pressure $p/2$. This is very nearly true if the lines are totally absorbed at their centres, which is the case, at least for the strong lines, over the pressure range which has been considered.

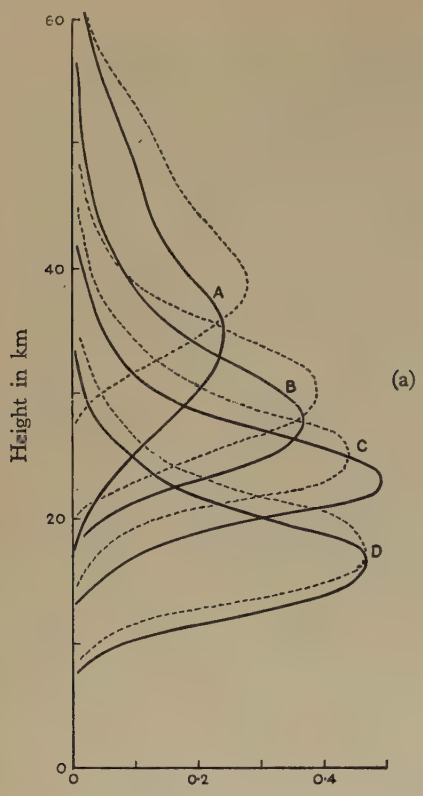

Figure 2 (a). Incremental transmission for 5 km layers.

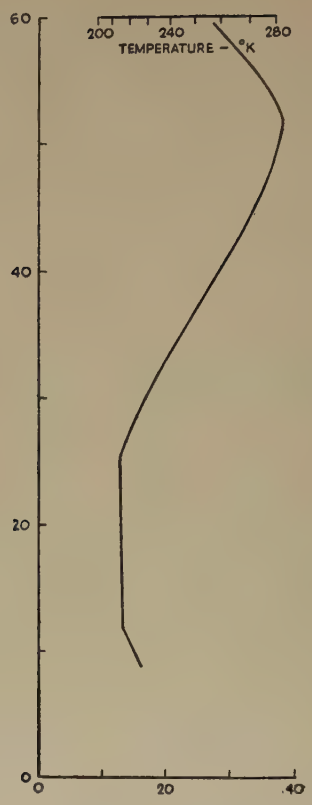

Figure 2 (b). Black-body emission at 15 μ in μ watts cm^{-2} wavenumber^{-1}.

Figure 2 (a). Incremental fractional transmission for 5 km layers of atmosphere as observed by an interferometer at the top of the atmosphere having a spacing between orders of 1·59 cm^{-1} and a band-width of transmission maxima of 0·1 cm^{-1}. The transmission maxima of the interferometer coincide with the line centres (curves *A*), are distant from line centres by 0·1 cm^{-1} (curves *B*), 0·2 cm^{-1} (curves *C*) and 0·4 cm^{-1} (curves *D*). The auxiliary filter is assumed to pass the region 679·0-680·5 cm^{-1} (dotted lines) i.e. one order only, and 669·7-693·7 cm^{-1} (full lines) i.e., 15 orders.

Figure 2 (b). Distribution of temperature with height for I.C.A.N. atmosphere, where the abscissa instead of being temperature is black-body energy at 15 μ. Temperatures are plotted on an auxiliary scale at the top of the diagram.

Fig. 2 (b) shows the distribution of temperature with height for the I.C.A.N. atmosphere where the abscissa, instead of being temperature, is black-body energy at 15 μ. Multiplication of the curves in Fig. 2 (a) with that in 2 (b) will thus give the distribution of the depth in the atmosphere from which the energy which reaches the detector originates, and the area between such curves and the *y*-axis will be the total energy incident on the detector.

If the calculations had been performed more exactly without using the simplifications listed above, the curves obtained would have been different in detail, though their general appearance and the potential discrimination in height which they illustrate, would not be affected

A group of curves having peaks at lower levels in the atmosphere may be obtained by choosing a wavelength region further from the band centre, say from 690-710 cm^{-1}. Radiation from a higher region – around 50 km – could be isolated by using a simple filter transmitting a band-width of about 1 cm^{-1} in the centre of the Q branch.

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Observations on the Aitken nuclei in Atlantic air

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(Manuscript received 20 June 1960; in revised form 23 August 1960)

SUMMARY

The concentration, size and distribution of charges on Aitken nuclei in air from the Atlantic Ocean were measured at a station on the western coast of Ireland where man-made pollution was most unlikely. The results are examined in relation to the meteorological elements and human activities. Some sources of the nuclei observed in maritime air are discussed.

1. INTRODUCTION

Some measurements of the concentration, size and distribution of charges on the Aitken nuclei in the air at a station on the western coast of Ireland were made during the summer months of 1958 and 1959. The station was situated on Mace Head, County Galway, (approx. 53° 20′ N, 9° 50′ W) and with winds from the sector between 170° to 300° from north it received air directly from the Atlantic Ocean which was uncontaminated by any local human activity. The effect of such activity on the concentration of Aitken nuclei is shown by the contrast with our results for other wind directions and from a series of observations made in Galway City. We also obtained evidence of the production of nuclei by natural processes in the littoral zone.

The primary purpose of our observations was an investigation of the ionization equilibrium in maritime air and a more complete description of our station, apparatus and method of working is given elsewhere (O'Connor and Sharkey 1960). The prevailing meteorological conditions and the state of the sea were recorded also for correlation with our other results.

2. TOTAL AITKEN NUCLEUS CONCENTRATION

The total Aitken nucleus concentration, Z, was measured with photoelectric nucleus counters as described by Pollak and O'Connor (1955). The value of Z recorded for each observation was the mean of five or ten individual measurements. The variation of Z with wind direction is given in Table 1.

TABLE 1. VARIATION OF NUCLEUS CONCENTRATION WITH WIND DIRECTION AT MACE HEAD

Wind direction	N	NE	E	SE	S	SW	W	NW
Concentration (nuclei per cm^3)	2,610	5,020	3,850	1,280	1,160	690	890	5,050
Number of observations	4	1	15	40	54	36	48	15

The frequency distribution of mean values of Z for different ranges of nucleus concentration are given in Table 2.

Almost 90 per cent of the observations gave Z less than 10,000 nuclei per cm^3. The individual values of the nucleus concentration ranged from a minimum of $Z = 64$ to a maximum of $Z = 212{,}000$. The median value was $Z = 1{,}120$ nuclei per cm^3.

TABLE 2. Frequency distribution of ranges of nucleus concentration at Mace Head

Concentration (nuclei per cm^3)	Less than 500	500 to 750	750 to 1,000	1,000 to 1,500	1,500 to 3,000	3,000 to 10,000	More than 10,000
Number of observations	26	28	37	36	31	31	24
Percentage of total	12	13	17	17	15	15	11

In general, the lowest steady concentrations occurred with southwest winds under conditions of prolonged outbreaks of rain or drizzle. Precipitation was usually accompanied by some decrease in Z but our results were not sufficient to isolate its effect from the other factors which can influence the concentration. Unlike Moore (1952) our results show no marked association between Z and the intensity of vertical mixing as judged by the prevailing cloud type.

In agreement with Moore (1952) and Hess (1951) we found no significant correlation between the nucleus concentration and either the wind speed or the roughness of the sea's surface and the amount of spray and surf along the coast and islands. This is not surprising as a certain minimum degree of violence in the disruption of a water surface is necessary for the production of Aitken nuclei (Nolan and O'Connor 1955). There were no winds stronger than Force 6 during the period of our observations but it does seem that sea spray is not a major source of Aitken nuclei.

Most of the higher values of Z occurred with winds in which man-made pollution could have been present. Those in the off-shore winds are primarily the result of human activity. These higher values of Z tended to fluctuate erratically. As the investigation of ionization equilibrium in maritime air was our primary interest and this required measurements with on-shore winds, and reasonably constant Z, our observations were mainly confined to such conditions.

Besides human activity we obtained strong evidence that there are some natural sources of nuclei in the littoral zone which can sometimes cause large-scale fluctuations in the nucleus concentration. This sporadic production of nuclei along the sea-shore has been known for a long time. It was described by Aitken (1911) who associated it with the action of sunshine. It was observed under cloudy conditions by Pollak and Murphy (1952) who state that the nuclei seemed to be formed on the foreshore which was undergoing drying between tides. We found that sunshine was not a necessary condition for their production as we observed them under completely overcast conditions and even after sunset. We also found that these nuclei were very small (radius less than $1{\cdot}1 \times 10^{-6}$ cm) and largely uncharged. This suggests that they are formed by some gaseous reaction and subsequent coagulation rather than the mechanical disruption of matter. They occurred during periods of very low tides and with winds from the direction of the only mud and sand banks in an otherwise very rocky shore-line. The work of Conway (1942), Verzar and Evans (1959) and Junge (1960) suggests that sulphur compounds from the littoral zone might be the source of these small nuclei. Further investigation will be necessary, however, to establish their origin.

Studies of the trajectories of the air masses arriving at Mace Head from the North Atlantic show that it is probable that many of the nuclei in this air have their origin in the European or American continents.

The marked effect of human activity on the nucleus content of the comparatively pure ocean air may be seen in the results of a series of over 300 observations of Z which were made at the station in the suburbs of Galway City (population 22,000) during the period December 1956 to May 1958. The percentage frequency distribution of values of Z in various ranges of nucleus concentration is given in Table 3.

TABLE 3. Frequency distribution of ranges of nucleus concentration in Galway City

Concentration range (nuclei per cm^3)	Less than 10,000	10,000 to 20,000	20,000 to 30,000	More than 30,000
Percentage of observations				
April-September	34	39	15	12
October-March	8	41	32	19
Whole year	20	40	24	16

As expected, the higher values of Z occurred with winds from the directions with the greater density of population, and the winter readings are higher than the corresponding summer readings. A general decrease in Z with wind speed was also observed. Only 34 per cent of the summer values were less than 10,000 nuclei per cm^3. The individual measurements of Z ranged from a minimum of $Z = 1{,}185$ to a maximum of $Z = 230{,}000$, with an average of $Z = 19{,}500$ nuclei per cm^3.

3. Distribution of charges on nuclei

The number of large ions, i.e., electrically charged Aitken nuclei in the atmosphere, may be found from the difference in the total nucleus concentration Z and the concentration of uncharged nuclei N_0. Simultaneous with our measurements of Z in maritime air at Mace Head we often used the photoelectric counters to determine N_0 in air that had been passed through a charged cylindrical condenser which removed all ions. The results, expressed as the ratio Z/N_0 and grouped according to Z are given in Table 4.

TABLE 4. Ratio of total to uncharged nuclei Z/N_0 in maritime air

	Range of nucleus concentration						
Z/N_0	Less than 500	500 to 750	750 to 1,000	1,000 to 1,500	1,500 to 3,000	More than 3,000	All values
Average values	2·26	1·93	1·59	1·49	1·33	1·28	1·72
Number of observations	14	19	21	19	4	4	81

4. Size of Aitken nuclei

The size of the Aitken nuclei in maritime air at Mace Head was determined from the measurement of their diffusion coefficient by the dynamic method of Nolan and Guerrini (1935), but taking into consideration the heterogeneity of the aerosol (Pollak and Metnieks 1959). Our results, expressed in units of 10^{-6} cm and grouped according to the nucleus concentration, are shown in Table 5.

TABLE 5. Radii of nuclei (10^{-6} cm) in maritime air grouped according to concentration

	Range of nucleus concentration						
Radius (10^{-6} cm)	Less than 500	500 to 750	750 to 1,000	1,000 to 1,500	1,500 to 3,000	More than 3,000	All values
Average values	4·05	2·97	2·30	2·50	1·99	–	2·74
Number of observations	7	12	14	13	5	–	51

We obtained simultaneous values of the relative humidity and the size of the nuclei on 23 occasions. The results are given in Table 6.

TABLE 6. Relative humidity and radius of nuclei (10^{-6} cm)

Relative humidity (per cent)	100-90	89-80	79-70	69-60
Radius, average value (10^{-6} cm)	2·96	3·62	2·89	2·33
Number of observations	7	8	6	2

5. Discussion of results

In considering the results given in Tables 4, 5 and 6 it should be remembered that they refer to maritime air only and that they were obtained mainly during the summer months. They do not include observations made when the nucleus concentration Z was fluctuating violently. On these occasions we found that Z/N_0 was practically unity and that most of the nuclei were lost in the diffusion apparatus. With these conditions it is difficult to get accurate measurements but it is clear that these nuclei are mainly uncharged and have an average radius of less than 10^{-6} cm.

A tendency for the average size of the nuclei to increase with decreasing nucleus concentration is evident from Table 5. The results in Table 6 show that the size of the nuclei is not greatly influenced by the relative humidity. These facts suggest that the nuclei are mainly small on production and subsequently grow by coagulation.

When Aitken nuclei and small ions remain together in the same air mass, a state of equilibrium charge distribution becomes established in which only a fraction of the nuclei acquire charges. This fraction, which is more usually expressed by the ratio Z/N_0, is a function of the size of the nuclei (Keefe, Nolan and Rich 1959). The increase in Z/N_0 with decreasing Z shown in Table 4 is probably due to the increase in the size of the nuclei and to the fact that the nuclei have had time to reach the equilibrium charge distribution appropriate to their size.

It is of interest to note from the figures in Table 4 that N, the number of large ions of one sign in maritime air (where $Z = 2N + N_0$) is approximately constant at 200 ions per cm^3.

The comparatively high values of the nucleus concentration in the non-maritime air at Mace Head, given in Table 1, are undoubtedly due to the influence of human activity. As such nuclei were not our primary interest, few observations were made on them. In general, they were larger than the nuclei in maritime air and a greater fraction of them was charged.

Acknowledgments

We have pleasure in thanking the Meteorological and Geophysical section of the Dublin Institute for Advanced Studies for the loan of some apparatus and Professor P. J. Nolan of University College, Dublin, for helpful discussion of our results.

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551.511.6 : 532.517.4 : 536.2

An alternative derivation of the diabatic wind profile

By H. A. PANOFSKY
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(Manuscript received 30 June 1960)

Summary

Ellison's wind profile is re-derived under the assumption that in neutral and unstable air the distance from the ground determines the eddy structure.

1. Introduction

The similarity theory has been successful in suggesting the form of the wind profile for forced and fully-free convection. From these limiting profiles, Ellison (1957) has designed an interpolation formula which fits the observed profiles over the whole range from forced to free convection (Panofsky, Blackadar and McVehil 1960). This theory makes it further possible to infer the heat flux from the wind profile. However, in stable air, Ellison's formulation seems to be less successful.

It is the purpose of this note to suggest an alternative derivation of Ellison's profile, which might throw some light on the reason why the theory does not apply under stable conditions, but which otherwise makes it possible to obtain the same results also obtained by similarity theory.

2. The basic assumptions

The kinematic eddy viscocity K is dimensionally a length times a velocity, characteristic of the state of turbulence. In unstable and neutral air, the eddies reach the ground, so that this length will be assumed to be equal to the height above the ground. The velocity will in some manner depend on the vertical eddy velocities and will be denoted by w. We have then :

$$K = A\,w\,z \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where A is an undetermined constant. In stable air, the eddies do not reach the ground, and therefore Eq. (1) does not apply.

From studies of the spectrum of the vertical velocity (Panofsky and McCormick 1960) it appears that the characteristics of the vertical velocity depend on three parameters : the height above the ground z, the rate at which mechanical turbulent energy is created, $\epsilon_1 = u^{*2}\,\partial V/\partial z$, and the rate at which convective turbulent energy is created, $\epsilon_2 = gH/c_p\rho T$. Here, u^* is the friction velocity $(\tau/\rho)^{\frac{1}{2}}$, where τ is the surface stress, V is the wind speed, H the vertical heat flux, g the acceleration of gravity, c_p the specific heat at constant pressure, ρ density, and T the temperature.

By dimensional reasoning then,

$$w = B\,[(\epsilon_1 + \gamma\,\epsilon_2)\,z]^{1/3} \quad . \quad . \quad . \quad . \quad (2)$$

Here B and γ are other constants. There are two reasons why γ may be larger than one : first, convective energy might go primarily into vertical motion whereas mechanical energy is concentrated into horizontal components; second, convection leads to eddies with longer periods than mechanical turbulence; and it is primarily the large eddies which accomplish the vertical transport of momentum (see Panofsky and McCormick 1952).

Combining Eqs. (1) and (2) we get

$$K = Cz^{4/3}\,(\epsilon_1 + \gamma\,\epsilon_2)^{1/3} \quad . \quad . \quad . \quad . \quad (3)$$

where C is another constant. This equation superficially resembles Heisenberg's expression :

$$K \sim l^{4/3}\,\epsilon^{1/3} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where ϵ is the rate of energy production and l the eddy size. Comparison of Eqs. (3) and (4) suggests that, in neutral and unstable air, z takes the place of l in the lower atmosphere.

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3. Neutral conditions

Under neutral conditions, ϵ_2 is zero. Then, substituting Eq. (3) into the conditions that $\tau = \rho K \partial V/\partial z$, and integrating, we obtain :

$$\frac{\partial V}{\partial z} = \frac{u^*}{C^{3/4} z} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This is the differential equation for the familiar logarithmic profile, and shows that C in Eq. (3) must be $k^{4/3}$, where k is von Karman's constant. Thus, the final expression for K is :

$$K = (kz)^{4/3} (\epsilon_1 + \gamma \epsilon_2)^{1/3} \quad . \quad . \quad . \quad . \quad (6)$$

4. Unstable stratification

On substituting Eq. (6) into the relation between stress and shear, and simplifying, we obtain :

$$S^4 + \gamma S^3 z/L = 1 \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Here S is a logarithmic wind shear, defined by $S = (kz/u^*) \partial V/\partial z$, and L is the Monin-Obukhov length, defined by $L = u^{*3} c_p \rho T/kg H$.

Since $z/L = - S Rf$, where Rf is the flux Richardson number, Eq. (7) is identical with Ellison's Eq. (11) (Ellison 1957), if $\gamma = 1/Rf_{\text{crit}}$.

Eq. (7) is a precise expression for the Monin-Obukhov ' universal ' function $S = \phi (z/L)$. It is a differential equation for the wind profile, which can be integrated numerically, given the heat flux, the roughness length and the stress. Alternatively, the profile can be found if the roughness and the wind and temperature at two heights are known. Conversely, given the wind profile, the vertical heat flux can in principle be determined. Eq. (7) has also been derived by Yamamoto (1959).

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551.584.4

Weather in wheat

By H. L. PENMAN and I. F. LONG

(Read at Summer Meeting, Dundee, 30 June 1960. See *Q.J.*, **86**, p. 16)

Dr. R. S. Scorer : Could Dr. Penman confirm that the transports he is considering are due to eddies and not to molecular diffusion ? This is presumably the case because he is appealing to the logarithmic profile in his arguments. Could he also say why he uses vapour pressure and not mixing ratio as a measure of humidity : the latter being the measure strictly appropriate to turbulent transport ?

Dr. R. W. Gloyne : I should like to raise a more general point. Dr. Penman commented that plants are wasteful in their use of natural resources (specifically water). A somewhat similar attitude was expressed at, I believe, the 1959 British Association meeting, where plants were said to be very inefficient utilizers of solar energy (say 1-2 per cent only, for photosynthesis).

It is true that the amount of water in the tissues of a plant is small compared with the amount of water in circulation – this applies equally to animals and humans – but I question an attitude that, put crudely, judges the efficiency of a biological process in terms of the amount of matter and energy ' fixed ' : circulation and exchange are essential and very characteristic of life.

An extreme, though improbable consequence of adopting this attitude could lead to appreciable changes in the environment available for the efficient functioning of the organism.

Professor F. A. Brooks (*communicated*) : This excellent essay, illuminating many phases of micro-meteorology, barely indicates the enormous number of man-hours required in analysing the physical and plant influences on heat and moisture exchange in the crop depth. It is to be hoped that such interesting results from one field of wheat will lead to more generous support of other basic research in the physical interpretation of micro-environment. It was my privilege to spend nearly six months with the authors when this paper was in manuscript, and I found that the precision and accuracy of their recording system is so high that there need be no question of the reliability of the sensing transducer. The supporting data available were so extensive that I had time only to pursue one minor aspect of the whole study – that of the temperature ripple in the early hours before dawn (see Fig. 3 of the paper). This has a period of about two hours. Their interpretation, assuming non-penetrating advection of warmer air above the crop, is intriguing because in California we had concluded that similar temperature oscillations observed in a citrus orchard (Brooks *et al.* 1952) were due to ventilation within the tree depth by periodic density currents (which were unstable in a time sense because of progressive decrease in air drag, with increase in degree of inversion stability). A third explanation, discussed below, was a possible periodicity in net radiation due to varying cloud cover, and a fourth explanation came from discussion at Rothamsted of the possible graduated penetration of the wheat stand by overhead wind gusts. In the time available to me for this study, none of the four possibilities could be proved as either dominant or negligible. The conclusion, therefore, is that all four factors may have contributed, and only when each can be expressed in an equation with known coefficients can the problem be put into an automatic computer and the proportions found which yield a best fit to the observations.

Some results of the study of the third possibility of periodicity in net radiation due to variable cloud cover may be of interest, because the weather records for that night indicate some partial cloudiness. The record of solar radiation for the morning of 22 June 1955 compared with a clear-sky record the previous morning show 20 to 40 per cent depletion by clouds in the east, lasting 45 min., beginning 40 min. after sunrise. This coincides with the observed pause in temperature rise near 05 hr.

Considering next a possible oscillation in net long-wave radiation for the earlier temperature cycle, rough calculations were made assuming a hexagonal pattern of scattered cylindrical clouds having a vertical thickness of 0·234 times the cloud spacing. For cloud bases which cover 0·2 of the horizontal plane, the open-sky angle factor was found to be 0·6 instead of 0·8, and if cloud-base area defines 0·6 cloud cover, the open sky angle factor is less than 0·2. Furthermore there is little difference whether the calculation spot on the ground is directly under the cloud centre or in the open between clouds, unless the height to cloud base is less than the cloud spacing. This means that if there had been a varying cloud cover before sunrise 22 June 1955, which produced a significant change in net long-wave radiation it must have been a short-time overcast coinciding with the first temperature rise observed.

The authors bring out very definitely the differences in horizontal distribution of temperature and humidity and these, we believe, are the main difficulties in defining advection. Because the evaporation problem warrants extensive research, including the effect of border trees, we have undertaken, here at Davis College, University of California, a cooperative project including determinations of the variation of heat and moisture balances with distance from the leading edge of an irrigated plot. We are taking pains to equal the degree of reliability in observations demonstrated at Rothamsted.

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Dr. H. L. PENMAN (*in reply*) : There is evidence in the paper (Tables 2 and 4) that near the ground inside the crop, the transport constant can be of the same order as the molecular coefficient ; elsewhere the constant is several orders of magnitude greater, and we follow the rest of the meteorological world in presuming that it is an eddy transfer coefficient. The form of the potential used in the transport equation is unimportant as long as the equation is self-consistent in units. We use vapour pressure because tables exist that make processing of wet- and dry-bulb temperature readings easy, the conversion from mixing ratio being explicit in the psychrometer constant.

Crude or not, the attitude questioned by Dr. Gloyne is one that an agricultural meteorologist must adopt in assessing the efficiency of plant growth. Dr. Gloyne need not worry unduly, even if we get to his extreme. The average water use by vegetation in Great Britain is about 400-500 mm $year^{-1}$: the energy used in photo-synthesis – in the same unit – is about 10-15 mm $year^{-1}$. If ever we can double the latter, i.e., double crop yields, the effect on the energy balance will be too slight to be measurable by present-day equipment.

Mr. Long and I are both grateful to Prof. Brooks for his kind remarks about the paper, and for the stimulating interest he took in the work on which it was based. As a supplement to the point of detail he discusses, Dr. Yabuki has informed us that Japanese workers have observed a similar air-temperature ripple with a period of about two hours.

551.510.41 : 551.584.42

The flux of CO_2 over a field of sugar beet

By J. L. MONTEITH and G. SZIECZ

(Read at Summer Meeting, Dundee, 30 June 1960. See *Q.J.*, **86**, p. 205)

Dr. R. S. SCORER : Dr. Monteith claimed for his instrument an accuracy which did not seem to be borne out by the traces. With the accuracy he claimed the two traces should be mirror-images of one another in the mean because his bottles (of which he showed a slide) smooth the samples over 20 min. and the readings are every minute or so.

It is very interesting to see this estimate of no sensible heat being transported upwards from a green crop. This is in accordance with glider-pilots' experiences that thermals rise predominantly from areas from which there is no transpiration of water. Professor Sheppard need not, therefore, be worried that in summer no heat appears to be rising from the ground – there are plenty of areas not covered by growing green vegetation, and in any case the green areas are giving up latent heat. Nor need Dr. Penman be too worried about the water lost in this way : it comes back as showers elsewhere.

Dr. R. C. RAINEY : As a biologist, I should like to emphasize the very great potential value, for field research in crop physiology, of such an experimental technique, which appears to be capable of providing not only day-to-day but even hour-by-hour estimates of net uptake of carbon dioxide. It is accordingly a little disappointing that the accuracy of these particular results should have been limited by the size of the half-acre plot studied.

For the record, it should perhaps be mentioned that the empirical formula of sucrose is $C_{12}\,H_{22}\,O_{11}$, not $C_6\,H_{12}\,O_6$ as stated by the authors, though this error does not affect their conclusions.

Mr. J. S. Sawyer : What proportion of the carbon dioxide in the atmosphere is taken up by vegetation in a day or a month ? Is there any large-scale problem as to how the CO_2 in the atmosphere is replaced ?

Dr. J. L. Monteith (*in reply*) : The amount of atmospheric carbon dioxide absorbed by a sugar-beet crop during the growing season may reach 0·25 g cm^{-2} ground surface, which is about half the total CO_2 content of a vertical column of air. In the northern hemisphere (but not in the southern) there is a seasonal variation in the CO_2 content of the free atmosphere, often in phase with the photosynthetic cycle but varying curiously from year to year (C. D. Keeling, *Tellus*, **12**, p. 200, 1960; W. Bischof, *Ibid.*, p. 216). The amplitude of the variation, about $\pm$ 10 p.p.m. from a mean of 320 p.p.m., is a measure of the efficiency of large-scale turbulence in transferring CO_2 from the oceans to land masses covered with active vegetation. On the micrometeorological scale, fluctuations in CO_2 content are much larger : in calm weather the concentration in air above a crop may drop to 250 p.p.m. during the day and rise to over 400 p.p.m. at night.

In reply to Dr. Scorer, the two traces will appear as perfect mirror images *only* when the ' zero ' signal of the amplifier is constant or varies linearly with time.

We are grateful to Dr. Rainey for his correction, and we can assure him that a much larger field has now been made available for CO_2 and other micrometeorological studies.

551.574.1 : 551.594.25 : 536.42

The fragmentation and electrification of freezing water drops

By B. J. MASON and J. MAYBANK

(Read 15 June 1960. See *Q.J.*, **86**, p. 176)

Dr. H. L. Penman : Can Dr. Mason tell us anything about the time factors in the re-distribution of charge ?

Dr. J. Hallett : I have seen spikes analogous to those described, growing from melt-water pools in a glacier after a night frost. The water is forced out at that point in the surface which remains liquid longest – usually at the junction of two thin ice crystals, growing in the surface.

The *rate* at which drops cool in the atmosphere might be expected to be much less than in these experiments. Will not the greater rate of cooling (1) cause the outside of the drop to be significantly colder than its centre, thus enabling an outer shell to form more easily; and (2) reduce the amount of air which can be absorbed by the drop before it is nucleated ?

Dr. J. M. Stagg (*President*) : Dr. Mason has given us an admirably clear summary of his important and delicate work with Mr. Maybank on the physics of what happens to individual water droplets when they freeze. My only doubt concerns the effect on the splintering process of suspending the drops from a vertical thermocouple or from a horizontal wire in the case of the smaller sizes. In either case would not the suspension necessarily produce a deformation and therefore an asymmetrical stress in the droplet ? If so, would not this deformation lead to a mode of disruption of freezing which need not be a replica of what would happen with a freely suspended drop ?

Dr. F. J. Scrase : Presumably the charges on the freezing drops which were measured by their deflection in an electric field were produced by splintering before the application of the field. Although such charging in the absence of the field does not appear to be of major importance in the electrification of thunderstorms the presence of a strong field in a cumulonimbus cloud might well cause considerably higher charges to be separated when the strongly polarized freezing drops splinter. Has Dr. Mason carried out any of his measurements in strong electric fields ?

Dr. W. Hitschfeld : The mechanism of electrification postulated by the authors in freezing water is analogous to the development of the so-called Thomson e.m.f. along electrical conductors in a temperature gradient. The Thomson e.m.f. is positive when the hot end is electrically positive relative to the cold end, and is accounted for, grossly, by the relative crowding of the free negative electrons into the colder end. The order of magnitude in common metals is some microvolts per degree.

In freezing water, the authors require some millivolts per degree, and on account of the greater mobility of the positive ions obtain an e.m.f. with a sense opposite to that usually prevailing in metals. In a fuller study of this phenomenon, full comparison with the metal phenomenon which – at least at ordinary temperatures – is now well understood, should be made.

Dr. R. S. Scorer : By doing experiments with de-aerated drops the authors have shown that dissolved gas apparently has some effect. But the effect might be simply to provide a cushion, without any effect on the mechanical properties of the ice shell. Or it might be to cause cracks to form so easily, where an air enclosure weakens the shell, that no substantial internal pressure is built up. But I would not have thought it would have made the shell "spongy" as Dr. Mason describes it, i.e. less rigid, but simply weaker at many places.

Dr. B. J. Mason (*in reply*) : In reply to Dr. Penman, a potential difference will arise across the frozen shell of the drop as long as there is a temperature gradient across it, and the magnitude of the separated charge will be proportional to the temperature difference. While the interior of the drop still contains some liquid a temperature gradient and therefore a potential gradient will exist and a splinter ejected during this stage will carry away a charge. When the drop has completely frozen and attained thermal equilibrium with its surroundings both the thermal and potential gradients disappear. The time taken for this condition to be reached will depend upon the drop size and the air temperature; for 1 mm-diameter drops freezing in air at − 15°C, the time required is about 200 sec.

Dr. Hallett's photographs of long spikes formed during the freezing of bulk water are a very spectacular example of the phenomenon we have described. Increasing the rate of cooling will certainly cause the ice shell to be formed more rapidly but the time required for formation of the *initial* shell will usually be so short compared with the total life time of the liquid droplet that I doubt whether this will greatly affect the quantity of air absorbed.

In agreement with Dr. Stagg, we were concerned about the effects of the support on the freezing behaviour of the drops. We investigated this and found that provided the drops were not grossly distorted from the spherical, the degree of splintering was not appreciably influenced by the shape of the drop and the nature of the support, and some experiments with freely-falling drops of 50-100 μ diameter gave very similar results to those obtained with supported drops.

In answer to Dr. Scrase, I confirm that the charges on the drops were measured in a field applied *after* splintering has occurred. We have not yet made similar measurements when splintering occurs in the presence of a strong field but are now preparing to do so.

Dr. Hitschfeld draws an analogy between my theory of charge separation in ice and the Thomson effect in metals. There are some similarities between the phenomena and, in this connexion, it is interesting to note that the mobility of protons in ice is comparable with that of electrons in some semi-conductors !

In reply to Dr. Scorer, I agree that the effect of the air may be simply to provide a cushion for the expansion or to cause the formation of cracks through which the liquid may ooze and so prevent the build-up of strong internal pressures. Such an explanation is given in Section 5 of our paper.

551.508.765 : 551.574.1

Observations of precipitation elements in cumulus clouds

By R. J. MURGATROYD and M. P. GARROD

(Read 15 June 1960. See *Q.J.*, **86**, p. 167)

Dr. R. S. Scorer : I think I detected a suggestion in this discussion that oceanic cumulus rain more easily because of something to do with nuclei. It is by no means obvious how droplets can grow large, quicker, simply because there are fewer of them. If it is by condensation, then a greater degree of supersaturation (i.e., a notable absence of sufficient nuclei) must be attained in oceanic cumulus. But this I don't believe to happen. If it is by coagulation, the suggestion would imply that in land cumulus the efficiency of catch is substantially reduced by the condensed water being spread among large numbers of very small droplets. This is a new suggestion – to prevent rain by adding nuclei (i.e., pollution), which again I do not accept. Surely the significant difference is in the updraught strength and the residence time of the droplets in the clouds, and is not this the only significant difference ?

It should go on record that rain or drizzle from small cumulus over the ocean was regularly observed by Coastal Command pilots during regular search flights during the last war, even when the cloud tops were much warmer than freezing.

Dr. B. J. Mason : The author's observation that non-supercooled clouds, formed in a maritime air mass, have a high probability of producing showers if their depth exceeds about 7,000 ft, is in accord with our radar observations. I should like to ask Dr. Murgatroyd whether he finds

any correlation between the concentration of large droplets in these clouds (and hence their tendency to precipitate) and the total concentration of cloud droplets. One might expect, for example, that in clean maritime air, the cloud will be composed of a lower concentration of rather larger droplets than a cloud of similar water content in highly polluted air, and that such a cloud will be more colloidally unstable. The low droplet concentration and consequently the larger mean size is, I am sure, an important factor in the release of showers from clouds little more than 1 km deep in places like Hawaii, although the magnitude and distribution of the updraught is also of great importance.

Mr. L. C. W. Bonacina : Yesterday morning, 14 June 1960, when the sky presented a magnificent pattern of thunder clouds I noticed a towering unglaciated cumulus penetrating the base of a glaciated cloud which appeared to be part of an anvil overhead. Very shortly afterwards this tower itself began to develop anvil shape with obvious signs of glaciating. Presumably this tower would have glaciated even if it had not met the said obstruction ; but I should like to ask the authors to what extent they think the process was accelerated.

Mr. J. R. Probert-Jones : Oiled and sooted slides have previously permitted sampling of cloud droplets up to radii of about 50 μ, and radar techniques enable us to study precipitation development when drops of $\frac{1}{4}$ mm radius or more are present. Cloud physicists will therefore welcome this instrument which Dr. Murgatroyd has described to us, which promises to fill in this gap at one of the most interesting stages of precipitation development. However, Dr. Murgatroyd's 50 μ radius drops will evaporate in a fall of 5 m through air at 60 per cent relative humidity, and I am sure that drops of $\frac{1}{2}$ to 1 mm radius must be present for precipitation to occur from cumulus clouds.

Dr. W. Hitschfeld : Following up Mr. Probert-Jones's remarks, I wonder whether the excellent instrument designed by the authors could be adapted to provide cut-offs at drop diameters greater than 100 μ. If this is easily possible, would they consider feasible a multiple-head instrument, with cut-offs at, say, 100, 600, 1,100, 1,600, 2,000 μ, the findings of which would yield a fairly complete precipitation size-spectrum ?

Dr. R. J. Murgatroyd (*in reply*) : With regard to Dr. Scorer's comments I think it is now quite well established, both theoretically and observationally, that the influence of nucleus population on the precipitation process may often be quite as important as that of updraught strength and residence time. Some authors (e.g., Twomey, S. and Squires, P., 1959, *Tellus*, **11**, p. 408) go further and conclude that variations of the cloud nucleus content of the air are primarily responsible for variations of cloud droplet concentrations and colloidal stability. It is probable that differences in the ability of clouds in continental and maritime air masses to produce rain can be explained in this way, whereas the greater ability of layer cloud compared with cumuliform cloud to produce rain in a given air mass is probably a matter of updraught strength and residence time.

Dr. Mason's remarks on droplet concentration and colloidal instability are, I think, in agreement with my reply above to Dr. Scorer's comments. The difficulty in the United Kingdom in observing differences between ' continental type ' cloud droplet populations and ' maritime type ' cloud droplet populations is that practically all of the air masses over this country have had some maritime influence. However, in some of our observations (e.g., Durbin, W. G., *Tellus*, 1959, **11**, p. 202 and Singleton, F. and Smith, D. J., *Quart. J. R. Met. Soc.*, **86**, p. 454) the characteristic types of cloud spectra have been shown in layer cloud to change from a narrow spectrum to a broader spectrum with fewer small droplets and less total concentration but more large droplets as the cloud thickness increases. In cumuliform clouds the picture is confused although Ludlam (*Nubila*, **1**, 1958) argues that this tendency can also be detected in Durbin's observations.

With regard to Mr. Bonacina's interesting observation, it is very likely that the glaciation of rising cumulus towers containing supercooled water will be accelerated when they come into contact with clouds of ice particles. Similarly the relatively small areas of supercooled cloud droplets frequently observed in frontal conditions may well be due to the effects of ' seeding ' from above from ice-crystal clouds. This effect has been demonstrated practically by dropping dry ice from aircraft into supercooled stratiform cloud layers when local glaciation followed by precipitation often results. However, the general problem of when and why natural clouds first begin to freeze is still not well understood and very much more work is still needed on the effect of ice particles and other freezing nuclei on the onset of glaciation.

As Mr. Probert-Jones suggests, the likelihood of precipitation from cumulus clouds reaching the surface will depend on evaporation below cloud base which includes consideration of environmental humidity and height of cloud base amongst other factors, and the fall of the particle out of the cloud will depend on its size relative to the local updraught. In this work we were more concerned, however, with the development of particles likely to lead to precipitation at a later stage and the factors mentioned by Mr. Probert-Jones will have to be studied subsequently.

Dr. Hitschfeld's suggestion is interesting and the limiting factor, as Mr. Probert-Jones has shown (*Nature*, 1960, **86**, p. 271), will be the volume of air to be sampled, e.g., for the larger particles existing in, say, concentrations of one per cubic metre the surface area of the instrument has to be in the order of one thousand times that of the present instrument to get the same resolution. The approach to obtaining ' cut-off ' sizes would be determined more by foil thickness than size of the instrument head as the collection efficiency for these sizes would be very near to unity. The present instrument, of course, records any impact with these larger drops but the results are limited by the comparatively small volume of air which is sampled.

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Aircraft observations of rain and drizzle from layer clouds

By F. SINGLETON

(Read 15 June 1960. See *Q.J.*, **86**, p. 195)

Mr. R. F. Jones : Could the smaller number of large drops in anticyclonic layers of cloud be ascribed possibly to the likelihood that any air entrained into the cloud from above would be much drier, owing to subsidence, in anticyclones than in warm sectors ?

Dr. R. S. Scorer : Mr. Singleton's anticyclonic situations are ones in which there was probably no convection from the ground – and this was surely the important fact, not that it was an anticyclonic situation. One can quite easily have drizzle or rain from stratocumulus in the Azores anticyclone when the air is cold enough for there to be convection from the sea surface.

Why did the author compare cloud thickness with cloud-base height ? Surely base height is irrelevant or almost so ? The important parameter is the saturation mixing ratio at the base because this determines how much liquid water will be available in a given cloud thickness.

Dr. B. J. Mason : I was very interested in Mr. Singleton's observations which are similar, in many respects, to those which Howorth and I published for similar clouds occurring over Northern Ireland. This is encouraging in that the simple relationships which exist between cloud thickness and height of cloud base, in the two cases, may be of use to the forecaster. However, I think that orographic effects, which influence the updraught in the cloud, may well limit their general validity. Of the alternatives which Mr. Singleton offers, I think that the time which the droplets spend in the cloud before being carried to the boundaries by turbulent diffusion is the most important factor in determining their growth. If the clouds persist for more than about two hours, a sufficient number of droplets may remain in the cloud, grow by condensation to 20 or 30 μ radius, and thereafter mainly by coalescence into drizzle – or rain drops, without the aid of giant salt nuclei.

Mr. W. G. Durbin : Diagrams such as that produced by Mr. Singleton are certainly useful to forecasters but each synoptic situation has to be dealt with on its merits. The important clue often lies in whether layer cloud of known thickness is seen to be producing precipitation on the synoptic chart. Considerations of this sort do not, as yet, involve such aspects of cloud physics as droplet size or concentration.

Mr. F. Singleton (*in reply*) : In reply to Mr. Jones, the effect he mentions could be of importance although several of the cyclonic situations were accompanied by clear dry air above the low cloud sheet.

Answering Dr. Scorer, I agree that saturation mixing ratio at cloud base is important and I did on my original working diagrams investigate the effect of cloud-base temperature. I could, however, find no temperature effect when comparing clouds of the same pressure-height and, therefore, came to the conclusion that the more important parameters were cloud base and thickness.

Regarding Dr. Scorer's remarks about convection not being present in anticyclonic situations, I think that in such circumstances the residence times of drops would be greater and consequently lead to greater likelihood of precipitation, which is not in accordance with the results obtained.

I wish to thank Dr. Mason for his interest in this work. Whilst agreeing that residence time of drops in any type of cloud is important in determining the size to which drops can grow, the effect of salt nuclei spectra in low clouds must not be overlooked and we must surely seek to explain our observations by means of combinations of the two mechanisms.

Regarding Mr. Durbin's comments I would suggest that to know *why* a cloud precipitates is a first step to forecasting that a cloud will or will not precipitate.

Proceedings at meetings of the Society

Ordinary Meeting, 19 October 1960

The following candidates were declared to have been elected by the Council during the summer recess :

Fellows

Adetoro, Joseph Eyitayo, B.A., Dip.Ed., F.R.G.S., Kiriji Memorial College, Igbajo, Via Ikirun, Nigeria.
Axford, David Norman, M.A., Lane End, Oaktree Lane, Tavistock, Devon.
Houghton, Leslie F., Haven, Fergione Way, Bergerliet, Cape, South Africa. (*reinstatement*; *elected* 1951, *resigned* 1955).
Smith, William Medhurst, M.B.E., 8 Henville Road, Bromley, Kent.
Tribble, David Thomas, 32 Stilecroft Gardens, North Wembley, Middx.

Foreign Members

Murcray, David Guy, B.S., M.S., 2464 So. Stuart Street, Denver 19, Colorado, U.S.A.
Roncali, Giorgio, Ufficio Centrale Di Meteorologia E Di Ecologia Agraria, Via Caravita 7/A, Italy.

Student Member

Murkett, Alan James, 35 Lichfield Avenue, Herford, Yorks.

Canadian Branch

Fellows

Bergeron, Jacques Charles Joseph, B.Sc., M.A., 4830 St. Charles Road, STE Genevieve, Quebec.
Bradley, James Henry Stobart, B.A., M.A., c/o 140 Pleasant Park, RO, Ottawa.
Danard, Maurice Beverley, B.A.Sc., M.A., 15-1930 Portage Avenue, Winnipeg 12, Manitoba.
Enns, Peter, B.Sc., 52-13th Street N-W, Portage La Prairie, Manitoba.
Gallaher, Donald Frederick, B.A.Sc., M.A., 141 Bedford Road, Toronto 5, Ontario.
Hasnain, Syed Badrul, M.Sc., 315 Bloor Street West, Toronto 5, Ontario.
Jones, Hilliard Howard, B.Sc., 189 Wallace Street, Campbellford, Ontario.
Maehara, Takeshi, B.A., R.R.2. Kelow NA, British Columbia.
Nissen, John William, B.A., M.A., 19 Canning Avenue, Toronto 6, Ontario.
Stewart, Ian Malcolm, B.Sc., 1588 Wolseley Avenue, Winnipeg 10, Manitoba.
Trenton, James Norman, B.A., M.A., 79 Elwood Boulevard, Toronto, Ontario.
Walkden, Raymond William, B.A., Box 334, Winnipeg, Manitoba.

The following candidates were then declared to have been elected that day by the Council :

Fellows

Drazin, Philip Gerald, Ph.D., Department of Mathematics, Queen's Building, University Walk, Bristol 8, Somerset.
Fowlis, William Webster, B.Sc., Department of Physics, King's College, Newcastle on Tyne, 1, Northumberland.
Hicks, Capt. Ralph Alan Woodhams, 55 Winchester Road, Andover, Hants.
Hide, Raymond, B.Sc., Ph.D., Department of Physics, King's College, Newcastle on Tyne, Northumberland.
Lynes, John Frederick, 11 Hampton Gardens, Prittlewell, Southend on Sea, Essex.

Student Members

Clough, Miss Anne, 2 Greenbank Crescent, Bassett, Southampton.
Linnard, John, 20 Victoria Avenue, Mumbles, Swansea, Glamorgan.
Dowson, Peter Leslie, 31 Reddington Drive, Langley, Slough, Bucks.

Foreign Members

Murphy, Allan Hunt, B.S., M.S., 1914 North Main Street, East Hartford 8, U.S.A.
Ta, Thu, B.Sc., M.A., Burma Meteorological Department, New Secretariat, Rangoon, Burma.

Canadian Branch

Fellows

Macloon, Frank C., B.Sc., New Brunswick Electric Power Commission, 527 King Street, Fredericton, New Brunswick.
Sampson, John Edward, B.A., 120 Ellerbeck Avenue, Kingston, Ontario.

Student Member

Okulaja, Feluso Olanrewaju, Department of Meteorology, McGill University, Montreal, P.Q.

The following papers were read and discussed :

1. ' Spectroscopic observations of the water-vapour content of the stratosphere ' by J. T. Houghton and J. S. Seeley (*Q.J.*, **86**, p. 358), presented by Dr. Seeley.
2. ' A direct measurement of the humidity in the stratosphere, using a cooled-vapour trap ' by F. R. Barclay, M. J. W. Elliott, J. V. Jelly and P. Goldsmith (*Q.J.*, **86**, p. 259), presented by Mr. Goldsmith.
3. ' A theory of fronts in relation to surface stress ' by F. K. Ball (*Q.J.*, **86**, p. 51).

Evening Discourse, 2 November 1960

The first of three Evening Discourses planned for the 1960-61 session took place in the Society's Rooms : Dr. A. W. Brewer spoke on ' The stratosphere.'

An account of the Discourse will appear in January issue of *Weather*.

Evening Lectures for sixth-form pupils, 25, 26 and 27 October 1960

Dr. T. W. Wormell, M.A., delivered the lectures in the second series of special evening lectures held in the Society's Rooms for invited audiences of sixth-form pupils and their science teachers. The lectures are intended for pupils who will be reading mathematics and/or physics at university. The title of the lecture was ' Electrical phenomena in the atmosphere ' and was intended to describe how the subject of electricity studied at school could be applied to the science of meteorology.

The lecture was well attended on each of the three evenings, and, in all, nearly 300 pupils and teachers were present.

An account of these lectures will appear in January issue of *Weather*.

We, representing the Council and members of the Royal Meteorological Society, desire to offer our Society's warmest congratulations to the Royal Society on the occasion of the tercentenary of its foundation.

Throughout its history the Royal Society has held a pre-eminent position in the promotion of natural knowledge, and we are deeply sensible of the beneficial influence it has exerted in the encouragement of scientific discovery and in the dissemination of authoritative scientific knowledge so that new advances and developments might be applied to the welfare of mankind.

We recognise with special satisfaction the support which the Royal Society has afforded to the science of meteorology through the Gassiot Committee and the Meteorological Committee, through its sponsorship of the British meteorological activities in the International Polar Years 1882/3 and 1932/3 and the International Geophysical Year 1957/8, and through the many close associations between the two Societies since the foundation of the Royal Meteorological Society over a hundred years ago.

We cherish the fact that many pioneers in the scientific study of meteorology such as Boyle, Hooke, Newton and Halley were among the early fellows of the Royal Society, and we are proud that in more recent times and to date the membership of our Society has included many who have been elected to the Fellowship of the Royal Society.

In extending our congratulations and sincere good wishes for the future, the Royal Meteorological Society has every confidence that the Royal Society will continue to uphold its fine tradition of leadership in the world of science which has been built up over the past three hundred years.

Given under the Seal of the Society, 15 July 1960

President

Secretaries

Facsimile copy of Congratulatory Address to the Royal Society on the occasion of the tercentenary of its foundation presented by the President, Council and Members of the Royal Meteorological Society.

Ordinary Meeting, 16 November 1960

The following papers were read and discussed :

1. ' Some recent changes in atmospheric electricity and their cause ' by K. H. Stewart (*Q.J.*, **86**, p. 399).

2. ' The diabatic wind profile ' by H. A. Panofsky, A. K. Blackadar and G. E. McVehil (*Q.J.*, **86**, p. 390) presented by Dr. Panofsky.

3. ' Measured radiative flux divergence near the ground at night ' by J. P. Funk (*Q.J.*, **86**, p. 382) presented by Dr. J. L. Monteith.

Popular Lectures

28 November 1960 : Mr. P. J. Meade, O.B.E., B.Sc., took the Chair for the first of the Society's two Popular Lectures for 1960-1961. The lecture was delivered by Dr. R. C. Rainey, Ph.D., B.Sc., A.R.C.S., M.I.Bio., who showed how studies of the atmosphere help in solving the problems of applied biology in Africa and the Middle East.

The lecture was delivered in the Conference Hall of the London County Council, by invitation of the Chief Education Officer.

29 November 1960 : Dr. A. G. Forsdyke, Ph.D., A.R.C.S., D.I.C., took the Chair when Dr. Rainey presented his lecture again in the Conference Hall, and it was attended on this occasion by pupils from schools in Buckinghamshire, Essex, Kent, Hertfordshire and Surrey, and also from East Ham, West Ham and Croydon.

The total attendance at the two lectures was about 900.

An account of the Popular Lecture will appear in February issue of *Weather*.

Weather forecasting for aeronautics. By J. J. George. New York and London Academic Press, 1960. Pp. ix, 673; 592 Figures; 43 Tables. $15.

Joseph J. George, of Eastern Airlines, is known to many meteorolgists as a pioneer in semi-objective forecasting methods. These methods make extensive use of the flow and temperature patterns in the middle troposphere. George sums up the general purpose very well when he writes on p. 133 : ' It has long been known that cyclones under strong wind currents aloft tend to deepen, but surprisingly little has been done to utilise this knowledge in any methodical way.' For instance he has shown that for maritime cyclones under open contours at 500 mb, a combination of the contour gradient across 1,000 miles measured over the centre of the low, and the maximum thermal gradient over 1,000 miles in the northwest quadrant of the low, gives a useful forecast of the modal and maximum amount of deepening in the following 24 hours. There is an arbitrary correction to be applied to lows in the eastern parts of oceans which suggests that further parameters are required for an adequate specification of the initial conditions.

Among other matters dealt with are : (1) Methods for forecasting the track and behaviour of cyclones over America which should interest British meteorologists concerned with extended forecasting, despite the very considerable scatter evident in much of the data. (2) Methods of predicting the track and speed of hurricanes using 500 mb flow patterns and 850-500 mb thickness charts. (3) A method of forecasting the movement of anticyclones over North America. There is more than the usual amount of scatter in the dependent data for these (e.g., in the zone of speed forecasts 900-1,000 miles/30 hours there are extreme values of 100 and 4,000 miles). One would not be surprised to find even more anomalous movements if the method were used over the eastern Atlantic or northwest Europe.

In the chapter on the movement of cold lows at 500 mb there is an interesting rule for determining when a surface low will be " captured " by the adjacent upper low. For surface lows centred 350-700 miles away from the 500 mb low at the time of forecast ' capturing ' is highly probable within 24 hours if an easily determined ' diffluence ratio ' at 500 mb exceeds 2·0.

The account of methods of objective precipitation and fog-clearance forecasting are informative, though in the former the network of upper-air stations over America allows an approach which would be difficult here. The chapter on forecasting 500 mb winds follows fairly familiar lines but there is some study of the effects of warm and cold advection on 500 mb trough structure and movement which may not be well enough known over here.

The last fifth of the book is taken up with local forecasting studies for a number of airports in America.

There is no attempt to interpret the rules in terms of dynamical theory, and this is probably wise in a book of this kind. It is not a book that could be given to a raw meteorological recruit in the expectation that it would do him or his forecasts much good. The production is of a high standard and there are surprisingly few misprints for such a complex text. There is a curious typographical ghost at the bottom of page 506 : the real form is at the bottom of page 509.

The index could well be more complete. For example, there is no mention of the capturing process described above or of ' diffluence ratio ' and no entry for ' hurricanes.'

Mr. George is to be congratulated on making the attempt to gather together and describe these extremely complex procedures. It makes a fairly long book, 673 pages, that is not easy to read. Many of the techniques need to be carried out on actual charts to make them clear, but that is no fault in a book so obviously addressed to professional forecasters.

M.K.M.

Physics of precipitation (*Geophysical Monograph No. 5*), Ed. by H. Weickmann (Proc. Cloud Physics Conf., Woods Hole, Mass., 3-5 June 1959). Washington, (Amer. Geophys, Union), 1960. Pp. xii, 435; many Figures and Plates. $12.50.

This handsomely-produced volume contains a large number of valuable contributions to the study of clouds and precipitation.

After the forewords, and a review of problems and methods of rainfall investigation by Professor Bergeron (written in his inimitable manner but with scant reference to modern radar techniques), the papers are divided into five sections. The first is titled ' Morphology of precipitation clouds and cloud systems '; it contains contributions discussing the control of precipitation and cloud systems by large-scale disturbances, and its dynamical prediction, and two short papers on the dynamics of severe convective storms. The intrusion of such work into a subject which has for too long been dominated by microphysics is a welcome sign of progress.

The second section includes very diverse topics concerned with precipitation, ranging from anvil-cloud formation as seen by radar, to snow-crystal shape as an indicator of atmospheric structure. The third section is called ' Fundamental precipitation processes ' and contains mainly articles concerned with the microphysics of freezing nuclei and the aggregation of droplets and crystals. The fourth section is titled ' Hail formation ' a subject which has become of increasing importance with the recent intensification of cumulonimbus studies. Some beautiful pictures of hailstone sections by Dr. List are matched by equally impressive aerial photographs by Dr. Cunningham of hailstorm clouds. The dynamics of such clouds once more comes under strong discussion.

The final section is on 'Artificial precipitation control,' and has papers of the familiar kind noting the inconclusive results of experiments in the atmosphere with silver iodide, besides a proposal for localising cumulonimbus with an artificial heat source : enthusiasm and frustration seem still equally balanced in this field.

A general impression of the volume is that it contains many interesting and instructive papers, which, despite the efforts of the editor, defy arrangement in any coherent order. The phenomena of clouds and rains are so complicated and range over so many scales that for some time to come we can be charmed by their kaleidoscopic fascination whilst waiting for the more sombre framework of a comprehensive science to emerge.

F.H.L.

Atti del Congresso Scientifico Internazionale Sulle Correnti a Getto (Jet Stream) e Ondulatorie. Centro di Volo a Vela del Politecnico de Torino, 1959. Pp. 144; 189 Figures. 2,000 lire.

Early in June 1959 a three-day international congress on jet streams and mountain waves was held in Turin, Italy. The report on this congress is now presented as a collection of 25 papers read during the sessions.

After an introduction by W. Georgii, the first section of the report comprises seven papers on jet streams. Papers by E. Chambers and K. Wood discuss civil aviation problems associated with jet streams and clear-air turbulence over the Atlantic, Europe and the Mediterranean, and E. R. Reiter of Innsbruck University presents a number of isotach charts to illustrate the operational value of keeping track of maximum wind levels. Other papers in the section discuss the transport of radioactive material at the operating levels of jet aircraft (H. F. Fischer), aspects of atmospheric electricity associated with jet streams (H. Israël) and jet stream dynamics (P. Raethjen).

The second and larger section of the report is devoted to papers on mountain airflow. Most of these papers describe observations or observational techniques used to study lee wave and low-level rotor flow over mountainous regions in France, Germany, Italy, Yugoslavia and Czechoslovakia. Authors including N. Gerbier, J. Förchtgott, G. A. Ferrari, M. Reinhardt and A. Gazzola, give up-to-date news of the research projects which they have already introduced in their earlier contributions to mountain airflow literature. Various theoretical aspects of lee waves and rotor flow are dealt with by R. S. Scorer, P. Queney and E. Palm.

Towards the end of the report two papers discuss the aerodynamical problems of high-level flight and, finally, a paper by C. Lesca of Turin proposes photogrammetric methods for the determination of the dimensions of clouds.

The organizers of the congress are to be congratulated on producing the papers in a nicely printed booklet, with clear, distinct diagrams and within a reasonably short time after the congress. Although the papers do not appear to include startlingly new ideas for the theoretician, they comprise a modest but useful survey of current operational interest and ideas on jet streams and mountain airflow problems.

The papers are written in English, French, German or Italian, according to the linguistic preference of their authors, and each summary has an Italian translation.

C.E.W.

Besprechung der Ergebnisse die Luftströmung über dem Tropischen und Sudlichen Atlantik, 'Meteor' expedition, 1925-27, XVI, Teil 1, 3. Lfg. By E. Kuhlbrodt. Berlin (Walter de Gruyter), 1960. Pp. 85; 40 Figures; 6 Plates; 11 Tables. 54 D.M.

This volume contains a discussion of the results of aerological observations comprising the data of the 'Meteor' expedition as well as ship observations collected by the *Deutsche Seewarte* until 1939. Altogether there are some 7,000 pilot-balloon ascents available from the tropical Atlantic, the statistical treatment of which has already been published in Vol. XVI, Pt. 1, Nos. 1 and 2.

Full use is made of the data and a large number of aspects, such as the zonal and meridional structures of the lower and upper troposphere, is presented in diagrams; there are even sufficient observations to allow a study at the tropopause level. A separate discussion deals with aerological observations from the Bathurst and Fernando Norhona coastal stations at either end of the main shipping route. Graphical representations of the constancy q of tropical winds, are given in the chapter dealing with seasonal variations.

The contribution in hand is a valuable reference for the climatologist interested in tropical regions and wil lattain even more significance in the study of world-wide circulations when similar data become available from tropical regions of Africa and South America.

Following the tradition of previous publications in the 'Metcor' series, this book exhibits carefully edited, clear diagrams and pleases the scientist's strained eye with a relatively large print.

H.T.M.

Das Eiszeitalter : Grundlinien einer Geologie des Quartärs, (Bd. 2, Europa, Vorderasien und Nordafrika im Eiszeitalter). By P. Woldstedt, Stuttgart (F. Enke). P. vii, 438; 125 Figures; 24 Tables; Bibliography; Index. 65 D.M.

This is the second volume of three in which one of the world's best-known authorities provides us with a very thorough review of the facts with regard to the Ice Age. The first volume, in which two chapters were given to the climate of the Ice Age, was reviewed by Dr. C. E. P. Brooks in this Journal (**82**, 1956, p. 117).

In this second volume we are given an up-to-date review of the evidence for the spread of the ice, its oscillations, and its retreat, and the succession of events thereafter, shown, for example, by the progress of vegetation, the characteristics of soils, and the development of river terraces and beaches. Twelve chapters cover Europe from the North Cape and the British Isles to the Balkans. In the thirteenth chapter we move to Asia Minor, Egypt and Libya, and in the final chapter to north-west Africa. Not as much attention is given to Scandinavia as might be expected, and practically none to Iceland; understandably the geological evidence for events throughout the regions peripheral to the ice, from Britain and France to Germany and Russia, takes up much of the book. Meteorologists will accordingly find in this volume little of direct concern to their own science; but if they are curious about the evidence they cannot but benefit from a perusal of a work such as this. Implicit in these volumes, or in the recent and even more massive compilation by Charlesworth, is a salutary reminder of the stupendous mass of detailed field investigation upon which our knowledge depends. The author does not embark on controversy, but, where it exists, the relevant references are given. Twenty-eight pages of closely-printed bibliography and a faultless index, coming at the end of the flowing, fully-referenced and compactly-written text, are in themselves impressive.

As a publication, this book is a pleasure to handle. It fulfils a long-standing European tradition, wherein scientific scholarship is fitly allied with unobtrusive excellence of workmanship, free from the wearisome double-column ponderousness with which of late years we have been assailed from overseas. Meteorologists are now flying high; yet strange though this exposition of the work of the field scientist may appear, the aeronomist will not fail to respect the logical arguments by which the changes in the characteristics of a boundary-layer have been deduced by his geological colleagues who have chosen to walk.

G.M.

Climatological Atlas of Hungary. Budapest (Hungarian Met. Inst.), 1960.

This atlas contains 130 charts most of which are on a scale of 1 : 1,250,000 and the remainder are half that scale.

The climatological charts are preceded by maps of topography, soil type and vegetation, and followed by charts of mean sunshine, cloudiness, fog, actual (unreduced) temperature, dates of first and last frost, frequencies of ' ice ' etc., days, vapour pressure, precipitation, evapotranspiration, snowfall and duration of snow cover, wind roses and mean isobars. Extremes of temperature are indicated by mean isotherms of warmest and coldest winter and summer months and precipitation extremes are indicated in a similar way. The mean monthly rainfall at a number of well-distributed stations is exhibited by pillar diagrams. An unusual but welcome feature is a set of synoptic charts for six typical weather situations. The divisions of the year for the various elements are selected to suit their natural variations. The periods covered by the elements vary somewhat, but for most it is 1901 to 1950.

The descriptive text is in German as well as Hungarian. The printing is excellent, the colours of areas between isopleths being tastefully selected. This atlas is recommended for study by future designers of atlases in other countries as well as students of the climate of Hungary.

G.A.B.

Ground water hydrology. By D. K. Todd. London (Chapman & Hall), 1959. Pp. xii, 336; 156 Figures: 23 Tables; 86s.

One wonders why John Wiley & Sons have sent this book across the Atlantic, and why their British associates have sent a copy to the Society. It was written for American hydrologists, and, with rare exceptions, the references are deliberately restricted to the American literature, one group of exceptions being in the first chapter where a few foreign books and general references are included. The absence of any British publication may be just, but surely the Dutch have put sufficient of their experience on paper to deserve an honourable mention ?

The meteorological content is trifling for an author who has taken a master's degree in the subject: the evaporation term in the water balance is dismissed in 14 lines, and elsewhere the meteorological titbits occur in an entertaining chapter on fluctuations in ground-water level. In addition to the obvious effects of rainfall and evaporation, movement can be produced by land subsidence, changes in atmospheric pressure, wind, tides – and by the passage of a railway train.

The book is about ground water as a source of supply, and is an engineer's book covering much that must be already familiar to its prospective buyers. Most of the writing is lucid, only occasionally lapsing into near-incomprehensibility (as in the definitions of porosity on p. 23), and the illustrative line diagrams are admirable in their appearance and clarity. Engineers may welcome the use of the positive sign in the Darcy flow equation, the conventional negative sign being discarded as ' unnecessary and often confusing for practical applications,' but the physicists will be happy to find that even Dr. Todd has to be conventional when he introduces velocity potential and the Laplace equation.

Interest quickens when the author gets on to less familiar ground. A chapter on quality includes specifications of water suitable for drinking, for industrial use, and for irrigation, the last being supplemented by a graded list of salt-tolerant plants. The soil physics and soil chemistry in this section are too sketchy to be useful even as guides to sources of more thorough treatments. Artificial recharge of aquifers deservedly gets a chapter to itself, and the book concludes with an account of the kinds of model used in laboratory studies of fluid flow in porous media.

H.L.P.

Obituary Notices

Mr. C. F. AITKEN

Mr. Charles Frederick Aitken, who was elected to Fellowship of the Society on 28 June in its centenary year, 1950, died at the age of 73 in February 1960. Apart from periods of active service during the wars of 1914-18 and 1939-45 he passed the whole of his working life in the timber trade. Mr. Aitken had long found weather study an absorbing hobby. The observations of temperature which he maintained over many years at an altitude of 950 ft above m.s.l. in his native Derbyshire were valuable as a record of thermal conditions in an upland region of the country still poorly represented by climatological stations.

E.L.H.

Dr. W. L. BALLS, C.M.G., O.B.E., F.R.S.

William Lawrence Balls died at the age of 77 in July 1960, after a lifetime devoted to the study of the growth of cotton. From 1904, when he left St. John's College, Cambridge, until 1947, when he retired, nearly all of his work was done in Egypt at the Giza Farm 'which is the Egyptian equivalent of Rothamsted in England.' His first activity was botanical, on the plant physiology and genetics of cotton, but gradually it took in more of soil physics and meteorology as it became obvious that management of irrigation was the most important single factor in getting a good yield. In one of his later books – *The yields of a crop* – he reached the somewhat depressing conclusion that over the decades there had been a steady worsening of the root environment of the crop, caused by inadequate drainage and rising water table. The effect was to nullify the benefits that modern methods of plant breeding, manuring, spacing, pest control and the like had produced since 1897.

Balls was elected a Fellow in 1935, about mid-way between his first Society publication on 'Meteorological conditions in a field crop' (*Q.J.* 1913), and his last, on 'Varied weather in a garden' (*Weather* 1960). In this phenological note he commented : 'The joint researches of a meteorologist, a plant physiologist, and an expert on horticulture might disentangle these relationships.' He was all three in one, and his lively mind several times brought him into fields of meteorological interest, so that his publications include papers on climate, on hygrometry, and on harmonic analysis and the measurement of periodicity, as well as a more numerous group that can be classified as agricultural meteorology.

In his own field he is numbered among the great, and all in the Society who knew him, or knew of him, will regret the passing of one who conferred distinction on the Society by his membership of it.

H.L.P.

Mr. H. H. BENTLEY

In 1905-06 the presidential chair of this Society was occupied by Mr. Richard Bentley, F.S.A., F.L.S., F.R.G.S., formerly Publisher in Ordinary to Queen Victoria and head of the great publishing house of Richard Bentley and Son, which produced the works of Wilkie Collins, Marie Corelli, Mrs. Henry Wood and other famous writers of the 19th century. By the recent death, at an advanced age, of Horatio Haliburton Bentley the Society lost the second member of this distinguished family to be numbered among the Fellows. Mr. Bentley was elected on 18 May 1898. After a period of service with Messrs. W. H. Allen and Co., Bedford, he joined the shipbuilders Messrs. Harland and Wolff, Belfast, in 1912 as chief engineer and manager of their then newly formed electrical manufacturing department. He was responsible for the design and production of d.c. generating plant, motors for engine-room and deck machinery, automatic and hand-operated control gear and other appliances for marine use. During his 31 fruitful years with Messrs. Harland and Wolff, Mr. Bentley introduced numerous improvements in electrical machinery which contributed materially to safety at sea.

E.L.H.

Mr. D. MILES

We have to record the sudden death, on 31 December 1959, of Mr. David Miles, a Student Member of the Society since March 1958. Mr. Miles was educated at Maidenhead Grammar School and on leaving in 1956 received an appointment as statistical computer for Grassland Research, Hurley. This position he retained until two months before he died. Meteorology had a strong fascination for him; one of his declared ambitions was eventually to make his mark on the science.

E.L.H.

Mr. J. H. ROGERS, B.Comm., (London)

Mr. John Henry Rogers, who had been employed by the Society as Accounting Assistant, died on 2 July 1960, at the age of 73. It was after his retirement from active employment that Mr. Rogers entered the service of the Society in a part-time capacity and, although his health was not good, his sudden death was unexpected and aroused great regret among all who knew him. Mr. Rogers was a man of wide attainments in banking, accountancy and economics. He had a long and outstanding career teaching these subjects and was much sought after as a private tutor. The Society was fortunate to have Mr. Rogers's services in an important branch of its day-to-day work, and he will be greatly missed.

P.J.M.

Corrigenda

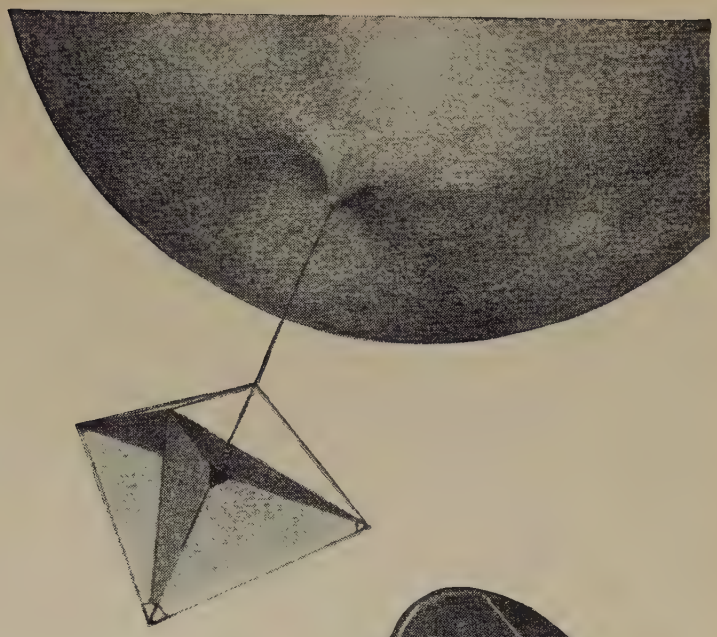
Vol. **86**, p. 400 (July 1960) Table I.

The last value in the second column (value of F for Lerwick for 1958) should read 135.

Vol. **86**, p. 558 (October 1960).

The following key should be added to the caption of Fig. 1 :

AC : Cache Lake, GW : Great Whale River, HO : Hopedale, HR : Harrington Harbour, IH : Indian House Lake, KL : Knob Lake, NI : Nitchequon, PH : Port Harrison, VP : Fort Chimo, YR : Goose Bay, ZV : Seven Islands.



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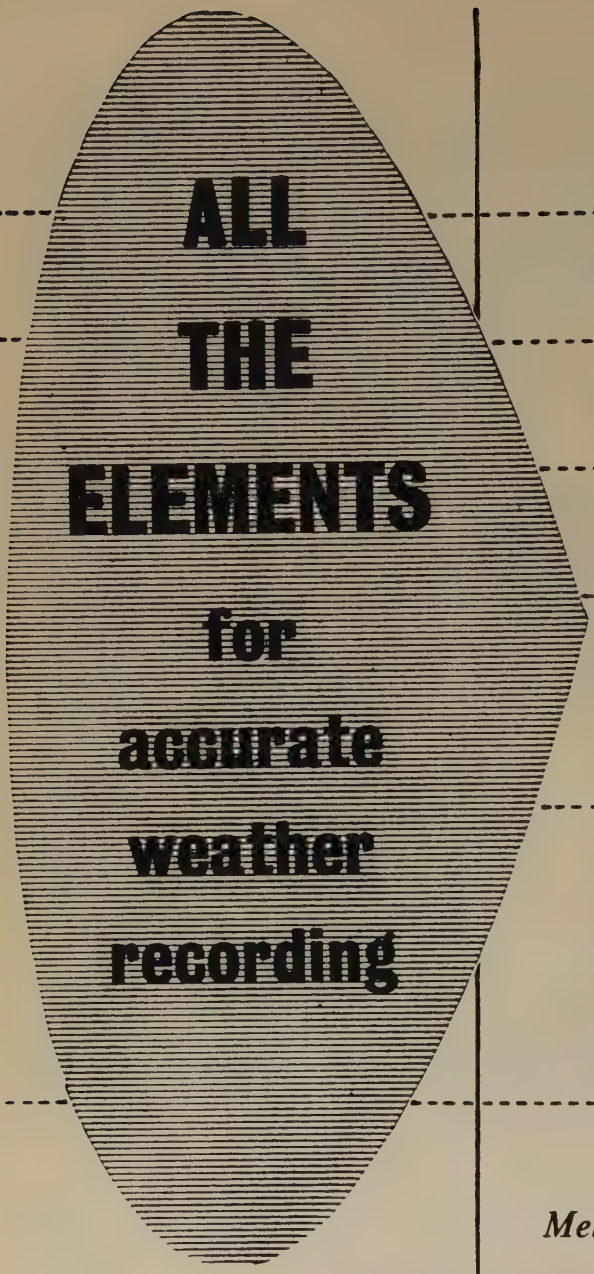

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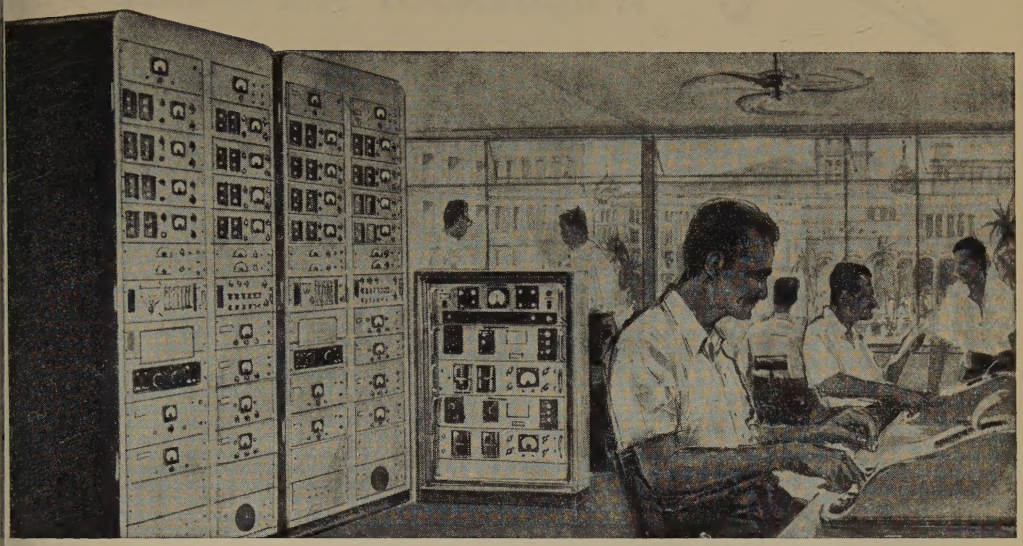

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